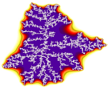

Nonlinear Plasma Physics: Treatment as a Complex System

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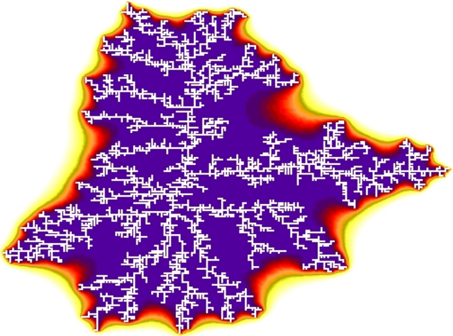
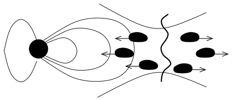


Outline

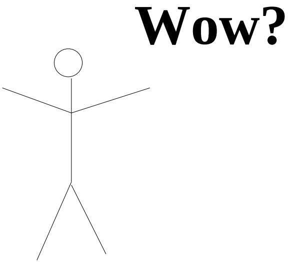
- What are complex systems
- Plasmas as complex systems
- Magnetosphere as a complex system
- Modeling
- Magnetosphere as a complex system



Mental Picture of intermittency

Complex system science

- Not a formal science
- Search for robust elements
- Emergence and self-organization



- Capacity to surprise (Casti 2006):

→Instabilities (macro from micro)

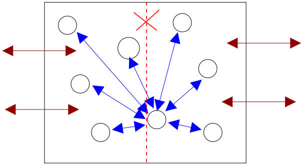
→Connectivity

→Emergence (self-organization)

What is a complex system

→generates surprise!

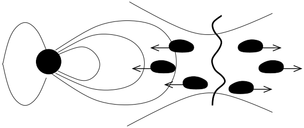
→many components, feedback, strong (nonlinear) interaction, open, hard to decompose into components, distributed (not centralized), self-organized, etc.



→regions of order and disorder

→times of order and disorder

→ Coherent structures



→city traffic

→wealth distribution

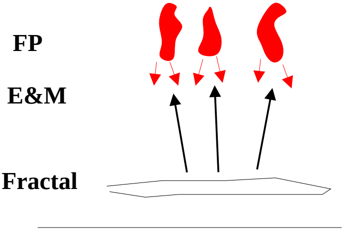
→stock market

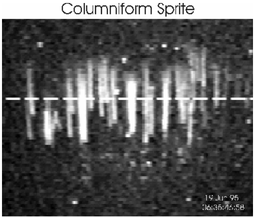
→river basins, sediment flow, etc.

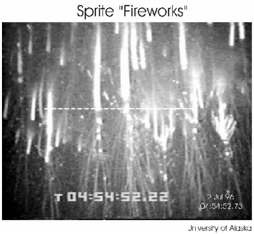
→etc., etc., etc.

Plasmas as complex systems I

→High altitude lighting
→Various nonlinear processes



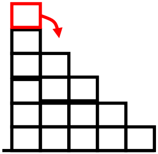




→Fractal and Multifractal
→Self-organization

Plasmas as complex systems II

Dissipation in solar flares
→Lu and Hamilton [1991]
→sandpiles
→spatio/temporal chaos



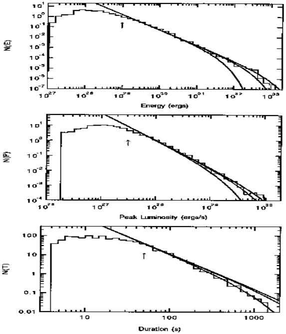


FIG. 8. Comparison between the frequency distributions of flares observed by *ISEE 3/ICE* (*histogram*) as a function of flare energy, peak luminosity, and duration with the theoretically predicted distributions (*solid lines*) for dimensionless ratios $L = 75$, 125, and 175. For $\Delta L = 400$ km, these values of L correspond to active region sizes 3×10^9 cm, 5×10^9 cm, and 7×10^9 cm, respectively. The observed distributions are incomplete approximately below the arrows.

→Dissipate magnetic flux
→Hysteresis!
→Global state!

Plasmas as complex systems III

→Boffeta et al.[1999]
→X-rays flares (waiting time)
→Shell models of MHD
→Reduced model of turbulence
→Include symmetries as

with $k_n = k_0 2^n$

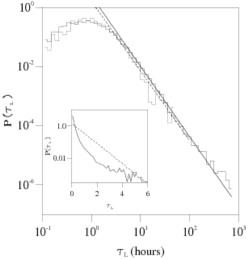


FIG. 1. Probability distribution of the laminar time $P(\tau_L)$ between two x-ray flares for data set A (dashed line) and data set B (full line). The straight lines are the respective power law fits. In the inset we show, in linear-log scale, the distribution for data set B (full line) and the distribution obtained through the SOC model (dashed line) which displays a clear exponential law. The variables shown in the inset have been normalized to the respective root-mean-square values.

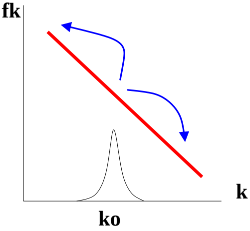
→Magnetic field

$$\frac{du_n}{dt} = \nu k_n^2 u_n + f_n + ik_n \left[(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - 1/4(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - 1/8(u_{n+2}u_{n-1} - b_{n+2}b_{n-1})\right]$$

$$\frac{db_n}{dt} = \eta k_n^2 b_n + ik_n \left[(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) + (u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + (u_{n+2}b_{n-1} - b_{n+2}u_{n-1})\right]/6$$

Plasmas as complex systems IV

→Turbulence in plasmas
→Feedack through magnetic field



→ORDER and DISORDER

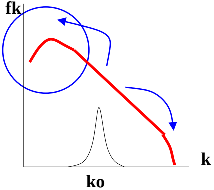
$$
\begin{aligned}
\frac{d\mu}{dt} &= -\mu\nabla\cdot\vec{U} \\
\mu\frac{d\vec{U}}{dt} &= \vec{J}\times\vec{B}-\nabla\bar{P}+\nu\nabla^2\vec{U} \\
\frac{d\bar{P}}{dt} &= -\gamma\bar{P}\nabla\cdot\vec{U}+(\gamma-1)\vec{J}\cdot(\vec{E}+\vec{U}\times\vec{B})-\nabla\bar{Q} \\
\frac{\partial\vec{B}}{\partial t} &= -\nabla\times\vec{E}
\end{aligned}
$$

→Ohm's law

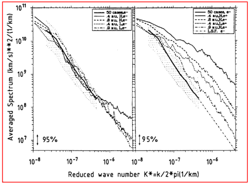
$$\vec{E}+\vec{U}\times\vec{B}=\eta\vec{J}+\alpha_1\vec{J}\times\vec{B}+\alpha_2\frac{\partial\vec{J}}{\partial t}+\alpha_3\nabla\bar{P}_e+\cdots$$

Plasmas as complex systems IV

Self-organization and turbulence?
→Global state
→Intermittent cascades
→ forward vs. inverse
→ Sandpiles?



•3D vs 2D (with and without Alfven effect)
→ Non isotropic
→ finite $\beta \to \nabla \cdot v \neq 0$
→ Multifractal



•How?
→ Spatio-temporal chaos vs stochastic?
→ Forced turbulence (shell models)?
→ Multifractal Brownian motion?
→ Others?

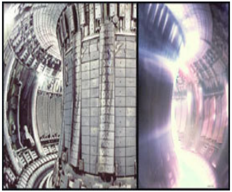
Elsasser variables
in the solar wind
Tu and Marsch
1990

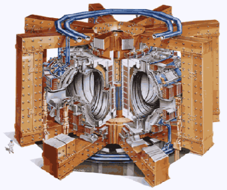
Plasmas as complex systems V

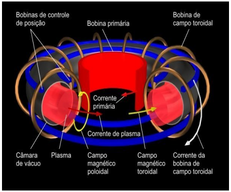
→Tokamaks (zonal flows)

→Transition L-H in Tokamaks
- → Suppress instabilities
- → Increase in velocity shear in convection
- → Turbulence
- → Transport barrier

→Improve magnetic confinement

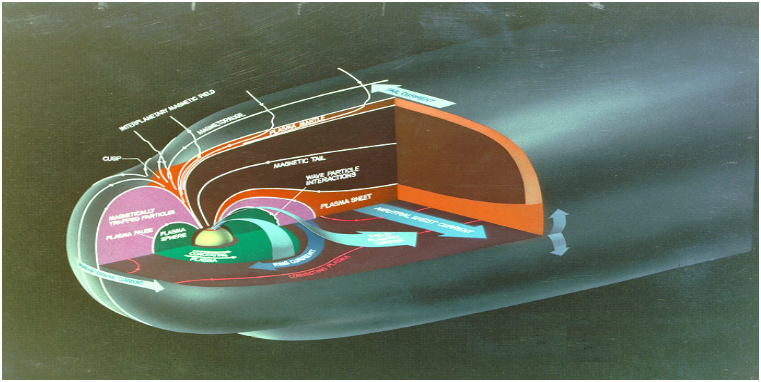






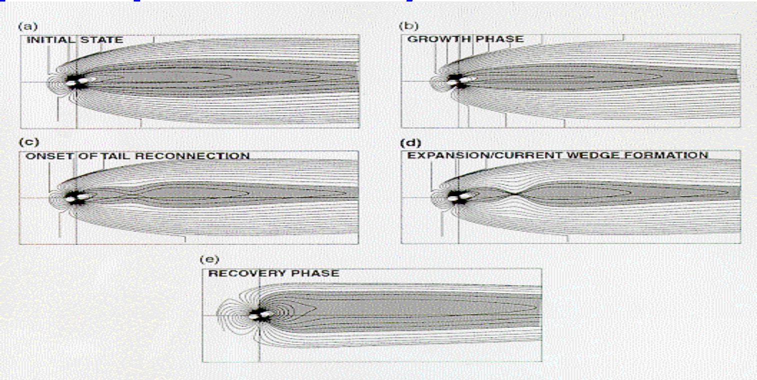
→Google images

Magnetosphere 0.0: Stationary state



Coherent and repetitive → substorms

Magnetosphere 1.0: Dynamics



→Near Earth Neutral Line Model

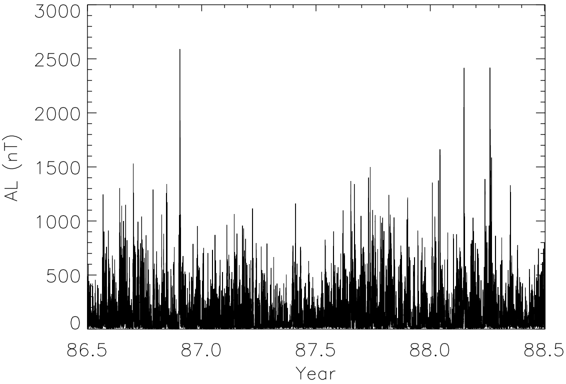
Dynamics (NENL) Baker et al.,1999; etc

Coherent and repetitive

MOVIE

Dynamics → low dimensionality

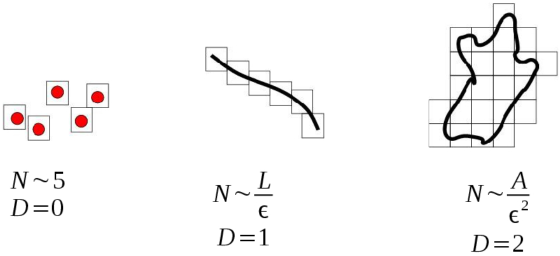
AL Index



Magnetospheric activity → AL, Au, AE, Dst, etc.
Low dimension?

Dynamics → low dimensionality

Dimension



Number of boxes required to cover structure

$$N(\epsilon) \sim \epsilon^{-D}$$

→as ϵ changes

Dynamics → low dimensionality

Length of England

$$\begin{aligned} d &= 200\ km \\ L &= 2400\ km \end{aligned}$$

$$\begin{aligned} d &= 100\ km \\ L &= 2800\ km \end{aligned}$$

$$\begin{aligned} d &= 50\ km \\ L &= 3400\ km \end{aligned}$$

→www.wikipedia.org

Number of lines required to cover structure

$$L(\epsilon) \sim d^{-(D-1)}$$

→as ϵ changes

Dynamics → low dimensionality

Embedding

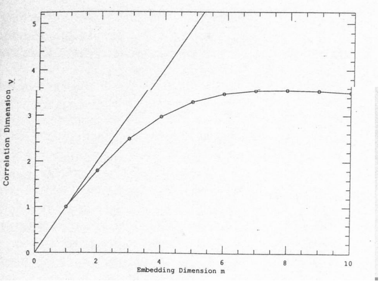
$$O(t) = [o(t), o(t-\tau), ..., o(t-(m-1)\tau)]$$

Box counting dimention

$$D_q(\epsilon) \sim \frac{\ln N(\epsilon)}{\ln \epsilon}$$

Multifractal

$$D_q(\epsilon) \sim \frac{1}{q-1} \frac{ln \sum_i \mu_i^q}{\ln \epsilon}$$



Sharma et al. GRL, 1999

Takens 1981, Lecture Notes in Mathematics, vol. 898. Springer-Verlag

But the system is driven by the solar wind

→As an input-output system

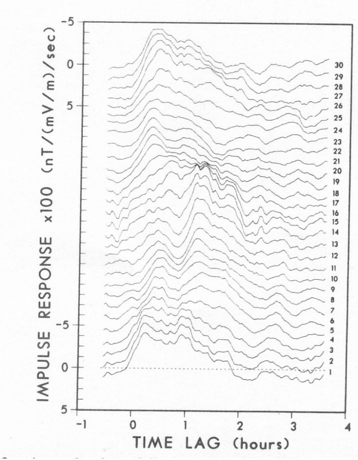
Space weather

Use linear filters

$$O(t) = \sum_{m=0}^{N} K_m I(t-m))$$

Different levels of solar wind $I = vB_s$

→Nonlinear Magnetospheric Response
→Use nonlinear models



Bargaze et al. JGR, 1985

Going to Physics

Low dimensional models (Nonlinear analogues)

Klimas et al., 1992
Faraday Loop Model

$$\frac{d\Phi}{d\tau} = (\epsilon_o - E)\sqrt{a}$$

$$\Phi = (1 + \gamma(\beta - 1)\left(\alpha\beta + \nu E + \frac{dE}{d\tau}\right)$$

$$\frac{d\beta}{d\tau} = \left[\begin{array}{ll} \epsilon_o & \Phi < \phi_c \\ \dot{\phi}_c & \Phi > \phi_c \end{array}\right.$$

Horton et al., 1992

$$L\frac{dI}{dt} = V_{sw} - V$$

$$C\frac{dV}{dt} = I - \alpha P^{1/2} - \Sigma V$$

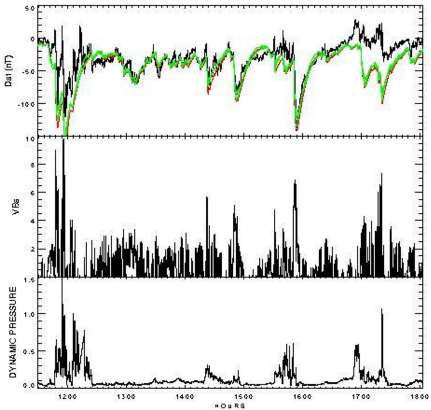
$$\frac{3}{2}\frac{dP}{dt} = \Sigma\frac{V^2}{\Omega} - u_o K_\parallel \theta(I - I$$

$$\frac{dK_\parallel}{dt} = \alpha P^{1/2} V - \frac{K_\parallel}{\tau_\parallel}$$

Space weather

Modeling and prediction

Singular value decomposition?
Linear vs Nonlinear models
Nearest neighbors
Fuzzy Logic
Neural nets



Applications
Indices: AL, AU, AE, PC, KP, Dst, Asym, etc.
From solar wind data

Nonlinear Filters

Vassiliadis et al., 1995
Valdivia et al., 1996, 1999, etc.

Space weather

Time delay embeddings

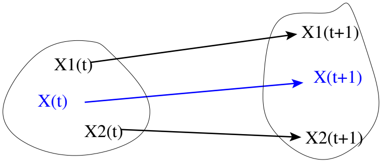
$$O(t) = [o(t), o(t-\tau_o), ..., o(t-m_o\tau)]$$

$$I(t) = [i(t), i(t-\tau_i), ..., i(t-m_i\tau)]$$

$$\vec{X}(t) = [O(t), I(t)]$$

Estimate (Phase space)

$$\boxed{\frac{d\vec{X}}{dt} = F[X]}$$

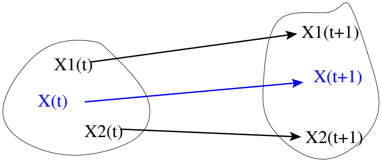


Space weather

Local prediction filters

$$o(t+1) = \frac{1}{N}\sum_{n=1}^{N} \bar{o}_n(t+1)$$

over the neighborhood of $\vec{X}$



Local linear aproximation

$$O(t+\Delta t) = \alpha + \sum_{n=0}^{m_o} K_n\, o(t-n) + \sum_{n=0}^{m_i} W_n\, i(t-n)$$

fit over the neighborhood of $\vec{X}$

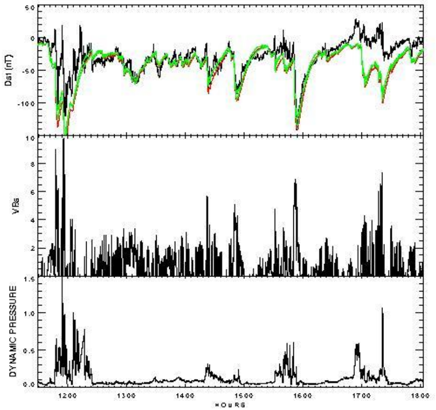
PHYSICS: Coefficients K_m and B_m can be transformed to physical parameters like time scales and coupling coefficients by transforming the above equation to a nonlinear ODE using Z-transforms

Local Linear Prediction Filters

Modeling
Imput $I = vB_s$

Output: Indices
AL, Au, AE, Dst, Pc, Kp, etc.

Try to relate to physics
of Ring current



Nonlinear Filters

Vassiliadis et al., 1995
Valdivia et al., 1996, 1999, etc.

Space weather

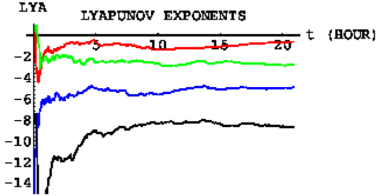
Go beyond! → Understand why

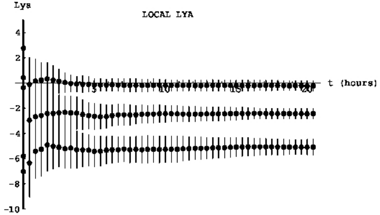
Synchronization driven system
Local Lyapunov Exponent

$$\frac{d\delta O}{dt} = DF[\vec{X}]\delta O$$

→Spectrum λ_n

Use local linear approximation





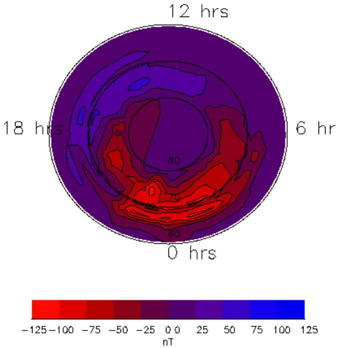
Space weather

Magnetospheric activity

Systems with spatial structure

High latitude magnetic perturbations

Average profile
→Harang discontinuity



Take simultaneous measurements from longitudinal chain
Image, Samba, Canopus, etc!

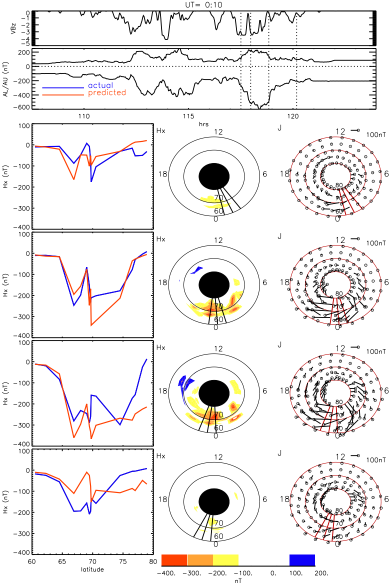
They rotate in local time → local lineal model for each local time

Space weather

Modeling high latitude
Image chain
Magnetic perturbations

Curren structure

Compare with actual measurements at local time



Nonlinear Filters

Vassiliadis et al., 1995
Valdivia et al., 1996, 1999, etc.

Space weather

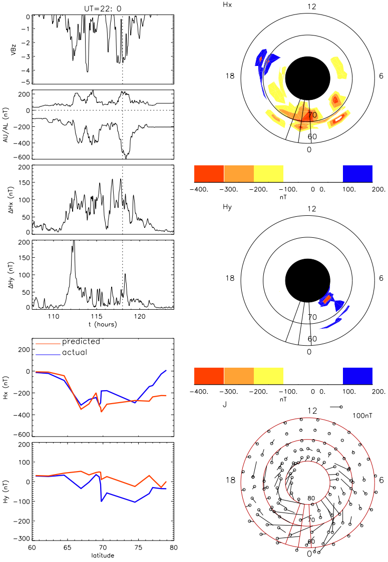
Modeling high latitude
Image chain
Magnetic perturbations

Curren structure

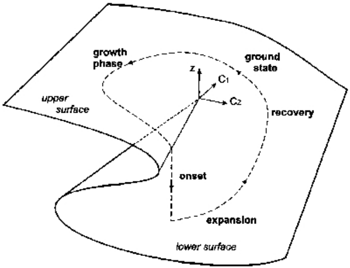
Compare with actual measurements at local time

Nonlinear Filters

Vassiliadis et al., 1995
Valdivia et al., 1996, 1999, etc.

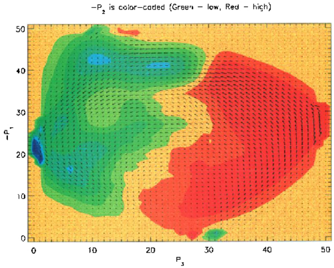


Space weather

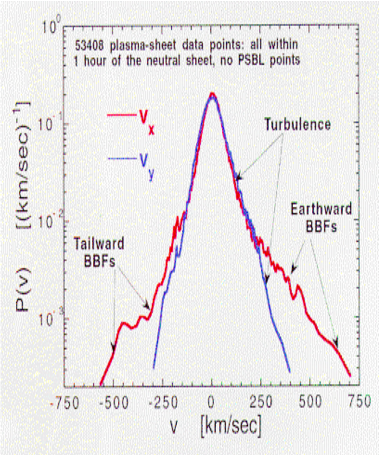


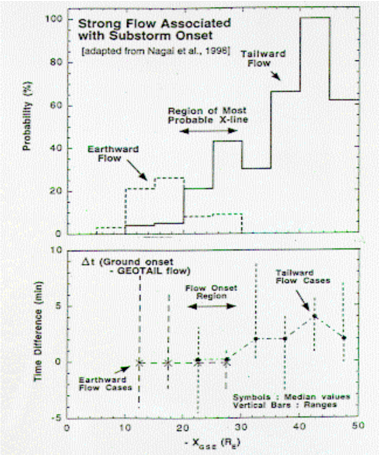
Phase transitions?
Catastrophe theory
Sitnov et al., JGR, 2000

Avalanching system??



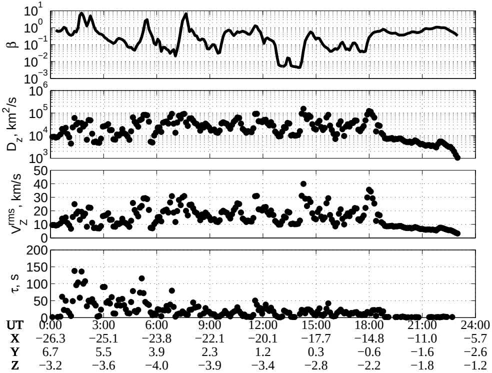
But ... Turbulence





Borovsky et al. 1997, Ohtani et al, 1998
Baumjohan et al., 1990, Angelopoulos et al., 1992, 1996, etc.

Turbulent Eddy Diffusion → current sheet



Flux

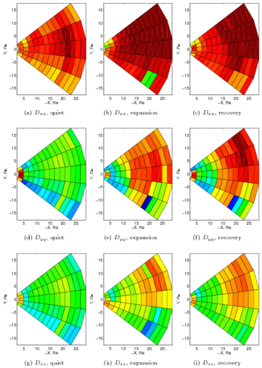
$$J = n < V > -D\nabla n$$

Diffusion coefficient

$$D_{\alpha\beta} = \frac{V^2_{rms,\alpha\beta}\ \tau_{\alpha\beta}}{2}$$

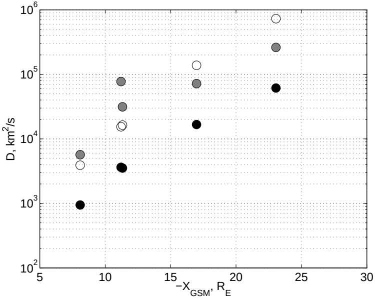
$$A_{\alpha\beta}(\tau) = \frac{\sum\left(V_\alpha(i) - \langle V_\alpha\rangle\right)\left(V_\beta(i+\tau) - \langle V_\beta\rangle\right)}{\sqrt{\sum\left(V_\alpha(i) - \langle V_\alpha\rangle\right)^2}\sqrt{\sum\left(V_\beta(i) - \langle V_\beta\rangle\right)^2}},$$

Turbulent Eddy Diffusion → current sheet



Spatial dependence of Diagonal elements of D from THEMIS

→Stepanova, Pinto, Valdivia, Antonova, JGR, 2011; JASTP 2011



Spatial dependence of diagonal elements of D using simultaneous measurements from THEMIS: D_{xx} (white circle), D_{yy} (gray circle), and D_{zz} (black circle)

Turbulent Eddy Diffusion

Why important? (Antonova et al, Stepanova et al.)
$D(B)$ can provide info about structure of the magnetotail

Flux $J = n < V > -D\nabla n$ in equilibrium

Assume Equilibrium Plasma Sheet - Lobe $J = 0$
$b = B/B_L$ normalized for magnetic field at Lobe

$$p + \frac{B^2}{8\pi} = const \qquad \rightarrow \qquad \frac{1}{p}\frac{dp}{dz} = f(b) = L\frac{V_z(b)}{D(b)}$$

Using $p = nk_B T$ only for ions since $T_i/T_e \approx 6$ and $L = (D/V_z)|_{B=B_L}$

Example: $D \sim B^{-2}$ and $V_z \sim B^{-1}$

$$f(b) \sim b \qquad \rightarrow \qquad p = p_o(1 - b^2)$$

Gives Harris sheet solution

$$\boxed{B = B_L \tanh(z/2L) \qquad \rightarrow \qquad p = p_o \cosh^{-2}(z/2L)}$$

Turbulent Eddy Diffusion

- anomalous diffusion

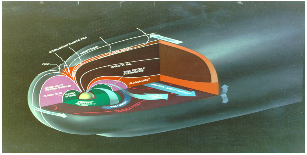
$$< \delta U_\alpha \delta U_\beta > \sim \tau^{h_{\alpha\beta}}$$

$$< \delta B_\alpha \delta B_\beta > \sim \tau^{\bar{h}_{\alpha\beta}}$$

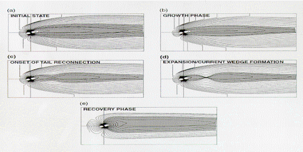
- super diffusion?
- sub-diffusion?

→ Fractional derivatives

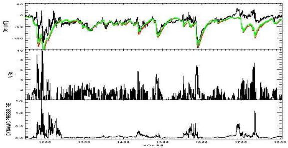
Magnetosphere 2.0: as a complex system



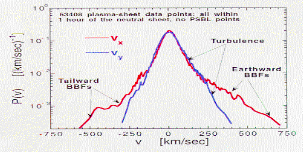
Static



Dynamics (NENL) Baker et al.,1999; etc
Coherent and repetitive



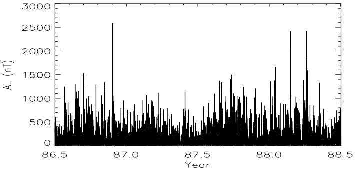
Nonlinear Filters
Vassiliadis et al., 1995
Valdivia et al., 1996, 1999, etc.

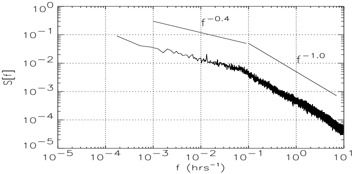


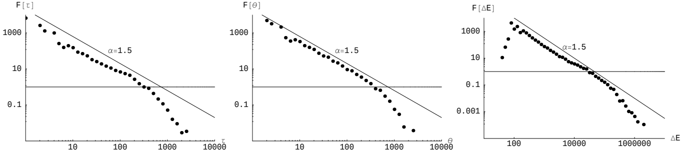
Turbulence and BBFs
Borovsky et al. 1997, Ohtani et al, 1998
Baumjohan et al., 1990, Angelopoulos et al., 1992, 1996, etc.

Temporal characterizations, e.g., AL

→Dissipation $\sim AL^2$ Consolini et al. 1997, 2000; Valdivia 2005, 2006; so on







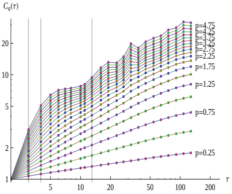
PDf of AL index. Power law with $\alpha \sim 1.5$

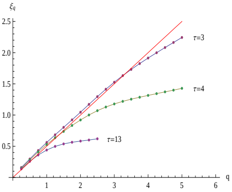
$$P[\alpha|\{\Delta E_i\}] \sim P[\alpha]P[\{\Delta E_i\}|\alpha] = P[\alpha]\Pi_i \frac{x_i^{-\alpha}}{\zeta(\alpha)} = P[\alpha]e^{-\alpha\sum \ln(\Delta E_i) - N\zeta(\alpha)}$$

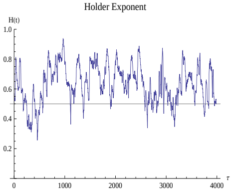
Self-organization $\rightarrow$ Multifractal in time

$\rightarrow$Estimate dissipation as $\sim AL^2$

$\rightarrow$Multifractal $\rightarrow$ intermittency (at different scales τ)







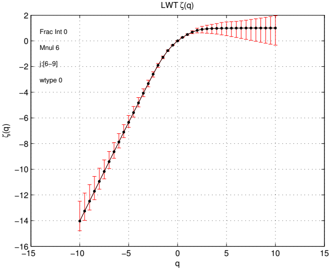
Holder exponent $\rightarrow$ $|AL^2(t) - AL^2(t+\tau)|^2 \sim \tau^{2h(t)}$

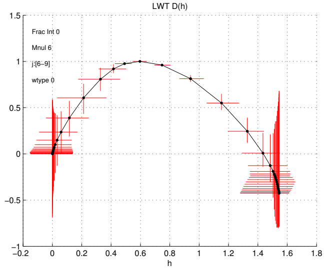
$\langle |AL^2(t) - AL(t+\tau)|^p \rangle \sim \tau^{\xi_p}$ $\rightarrow$ D(h)

$\rightarrow$Study with Shell model of reconnection (as in turbulence)

Wavelets → Multifractal in time

→Toledo et al. → energy conserving wavelets ($q < 0$)





→Multifractal dimension with associated error

Intermittency → Multi-fractal in time

•Use wavelet analysis (Voros et al. [2006, 2007, 2008, 2010])

$$P(f) \sim f^{-\alpha(t)}$$

•spectral breaks
•BBFs times
→ Large Scale ($0.7 - 5$ s) $\alpha \sim 2.6$
→ Small scale ($0.08 - 0.3$) $\alpha \sim 2.6$
•Non BBFs times
→ Large Scale ($0.7 - 5$ s) $\alpha \sim 1.7$
→ Small scale ($0.08 - 0.3$ s) absent

→ Kraichnan scaling (anisotropy $k_\perp$ and $k_\parallel$)

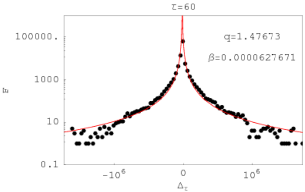
Intermittency → Multi-fractal in time

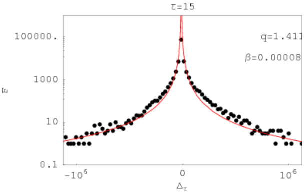
- $f < 0.01 - 0.08$ Hz ($100 - 13$ s) $\rightarrow \alpha \sim 0.5 - 1.5$
- $f < 1$ Hz (1 s) $\rightarrow \alpha \sim 1.7 - 3$
- $f > 1$ Hz (1 s) $\rightarrow \alpha \sim 3$ (Dissipation and Kinetic effects)

→physical regimes (larmor radious, Hall physics?)
→estimates Reynolds number $R_\nu \sim 1600$
not fully developed turbulence (sheet height restriction)

→nonlinear waves interacting

Non-extensive physics?





Tsallis et. al, as in turbulence with $\Delta_\tau(t) = AL^2(t+\tau) - AL^2(t)$

$$S_q = k\frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad P_q(x) = \frac{1}{Z_q}\left[1 - (1-q)\beta(x - \bar{x}_q)\right]^{\dfrac{1}{1-q}}$$

→Two distributions?

→Fisher's information?

Intermittency → Multi-fractal in space (ROMA)

Self-organization → spatio-temporal

- Lui et al., 2000; Uritsky et al., 2003; so on

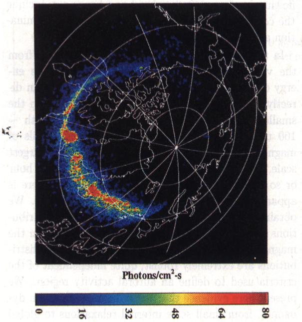


Figure 1. A sample of global auroral distribution from the Polar UV Imager.

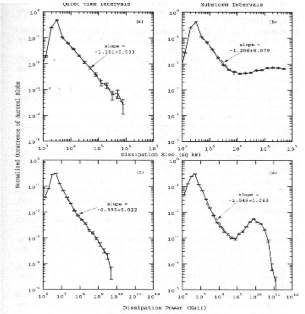
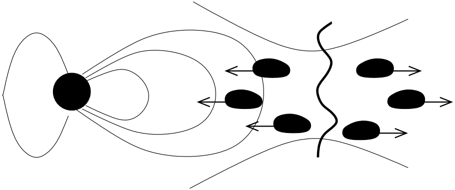


Figure 3. Probability distribution of size and power output of individual auroral region. (a) size distribution during quiet times. (b) size distribution during substorms. (c) power distribution during quiet times. (d) power distribution during substorms.

→BBFs and intermittence → Angelopoulos et al., 1999; so on

→Spatio-temporal dynamics

Reconcile seemingly contradicting observations?



Mental picture of intermittency Angelopoulos et al., 2000

→How to model?

→Sandpiles

- Chapman et al., 1998; Vassiliadis et al., 1998; Takalo et al., 1999; so on

→Physics-based

- Chang 1992; Milovanov et al., 1996; Klimas et al., 2000; Chang et al., 2002; Valdivia et al., 2003; so on

- Spatio-temporal chaos?
- Pure driven turbulence (shell models)?
- Multi-fractal Brownian motion?
- Others?

Self-organization and Turbulence?

→Global state

→Intermittent cascade Process → forward vs. inverse

- 3D vs 2D (with/without Alfven effect)
- Plasma sheet more complicated
- Non-isotropic
- Finite $\beta \to \nabla \cdot v \neq 0$
- Multi-fractal

→How?

- Spatio-temporal chaos?
- Pure driven turbulence (shell models)?
- Multi-fractal Brownian motion?
- Others?

Complex system and plasma

$$\frac{d\mu}{dt} = -\mu\nabla\cdot\vec{U}$$

$$\mu\frac{d\vec{U}}{dt} = \vec{J}\times\vec{B} - \nabla\bar{P} + \nu\nabla^2\vec{U}$$

$$\frac{d\bar{P}}{dt} = -\gamma\bar{P}\nabla\cdot\vec{U} + (\gamma-1)\vec{J}\cdot(\vec{E}+\vec{U}\times\vec{B}) - \nabla\bar{Q}$$

$$\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E}$$

Ohm's law $\vec{E}+\vec{U}\times\vec{B} = \eta\vec{J} + \alpha_1\vec{J}\times\vec{B} + \alpha_2\frac{\partial\vec{J}}{\partial t} + \alpha_3\nabla\bar{P}_e + ...$

Renormalized $\vec{E}+\vec{U}\times\vec{B} = \bar{\eta}[J, ...]\vec{J}...$

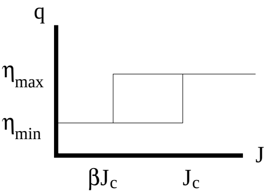
Intermittent dissipation

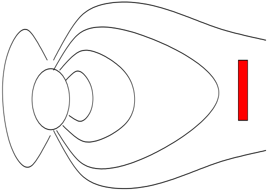
$A_y(z,t)$ and $J = -\nabla^2 A$

$$\frac{\partial A_y(z,t)}{\partial t} = \eta \frac{\partial^2 A_y(z,t)}{\partial z^2} + S_0 S(z,t)$$

$$\frac{d\eta}{dt} = \frac{q(J) - \eta}{\tau}$$

→Charge separation
→Nonlinear microphysics
→Similar to Lu 1995 continuous model





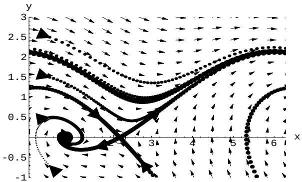
Hysteresis for q

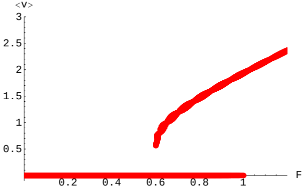
→Shell-like model of intermittent reconnection

→Klimas et al. (JGR 2000); Valdivia et al. (SSR 2003, ASR 2005; SSR 2006 ;ASR 2013); Uritsky et al. (JASTP 2001); Takalo et al (GRL 1999, JASTP 2001)

Hysteresis and η

$$\ddot{x} + \beta\dot{x} + \omega_o^2 sin(x) = E \quad \rightarrow \quad \frac{dv}{dt} \sim -\nabla \cdot P + \vec{v} \times \vec{B} + \delta\vec{E}$$





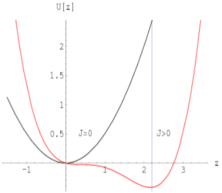
Fractional Fokker Planck

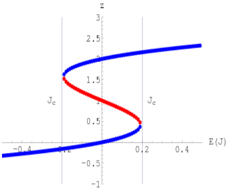
$$\frac{\partial A}{\partial t} = \partial_t^{1-\alpha}\nabla\left[D_\alpha \nabla A\right]$$

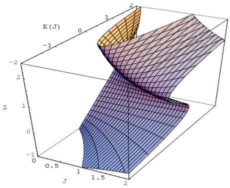
Hysteresis and η

- Transport coefficients converge faster in BKGY
- EM field with fluctuations $r_L \frac{\nabla B}{B} > 1$ and $\Omega \frac{\partial B}{\partial t} \frac{1}{B}$
- Hysteresis common in plasmas, automata, shell models, etc.
- Tangent bifurcation in plasma equations $\rightarrow E(J)$

$\ddot{x} + \dot{x} \times \bar{B}(z) = \bar{E}$ $\qquad \nabla \cdot \bar{E} = 4\pi\bar{\rho}$ $\qquad B(z, t = 0) = (B_o + Jz)\hat{x}$







Approximation 1D → hysteresis

•BKGY transport coefficient converge?

•Fluctuations E&M $r_L \frac{\nabla B}{B} > 1$, $\Omega \frac{\partial B}{\partial t} \frac{1}{B}$

•Hysteresis is common
→ plasmas, materials, automata, shell models, discharges, etc.

→Intermittent magnetic dissipation
→Loading-unloading
→η renormalizes other terms in Ohm's law
→Similar to Lu 1995 continuous model
→Hysteresis common in nature
→Charge separation
→Nonlinear microphysics

Model 1D: numerical integration

- 1-D more manageable model → discrete form

$$\frac{dA_n}{dt} = \frac{\eta_n}{\Delta z^2}(A_{n+1} + A_{n-1} - 2A_n) + S_n \qquad \frac{d\eta_n}{dt} = \frac{q_n - \eta_n}{\tau}$$

$$J(L) = 0 \qquad \left.\frac{\partial A}{\partial z}\right|_0 = 0 \qquad \Delta z = \frac{L}{N} \qquad S_n = S_0 \cos\left(\frac{\pi z}{2L}\right)$$

- Spatio-temporal chaos
  - → Shell model of intermittent reconnection
  - → Relevance of numerical integration
- Experimental estimation
  - Hysteresis (loading-unloading) → $\Delta E \sim \frac{2}{3} L^3 J_c (1 - \beta^2)$
  - η_{max}, η_{min} or even tensor (Stepanova et al, 2005)

Model 1D: simulation

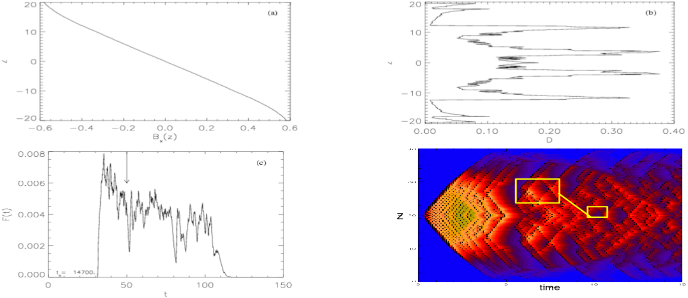
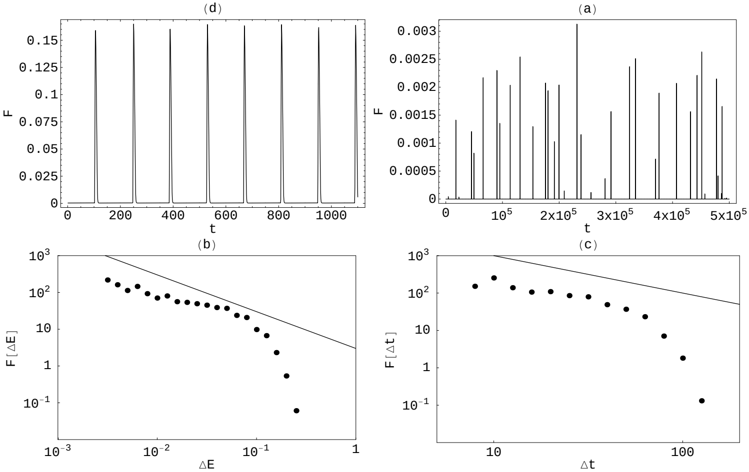


Figure: (a) B_x and (b) η for $\beta = 0.9$ (at time defined by arrow in (c)). (c) F(t). (d) ηJ^2

$$F(t) = \int \eta(x,t) J(x,t)^2 dx$$

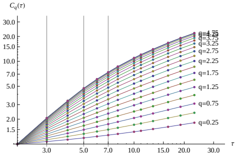
MOVIE

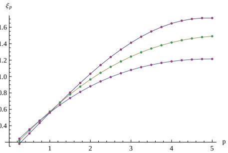
Approximation 1D: event statistics

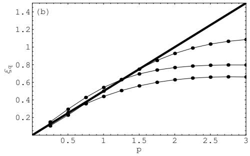


→Similar to $F(\Delta t)$ Angelopoulos et al. 2000

Model 1D → multifractal



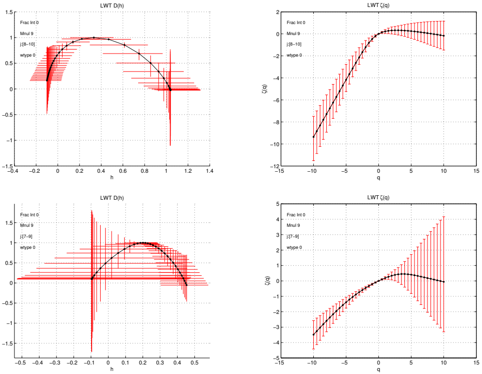




→Temporal Intermittency
→Spatial Intermittency
→nonlinear intermittent behavior

Model 1D → multifractal

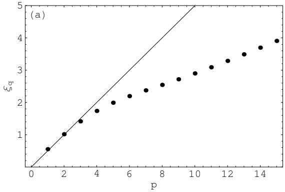
Wavelets



→Temporal Intermittency → similar to AL

Model 1d: Intermittent dissipation

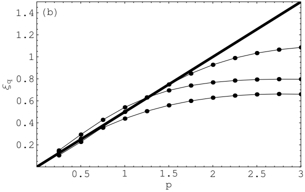
→Temporal intermittency



$$F(t) = \int \eta(x,t) J(x,t)^2 dx$$

$$\langle |F(t) - F(t+\tau)|^p \rangle \sim \tau^{\xi_p}$$

→Spatial intermittency

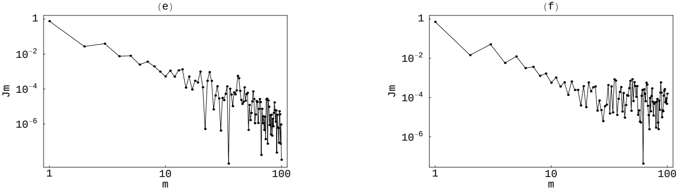


$$f(x,t) = \eta(x,t) J(x,t)^2$$

$$\langle |f(x+\Delta x) - f(x,t)|^p \rangle \sim \Delta x^{\xi_p(t)}$$

Model 1D: Cascading → self-organization

$$A(x,t) = \sum_{m=1}^{n_z} A_m(t) cos\left[\frac{(2m+1)\pi z}{2L}\right] + B_m(t) sin\left[\frac{(2m)\pi z}{2L}\right]$$



- Energy cascade

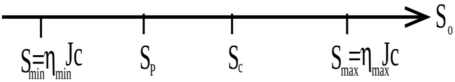
  → Self-organization

  → Scale-free dissipation

Bifurcation diagram

Controlled by

$\rightarrow \beta$ and η_{min}



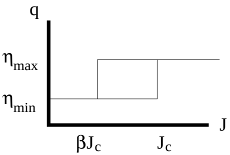


Figure: Hysteresis cycle

→Transition to spatio-temporal chaos

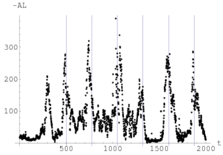
•temporal → periodic-quasiperiodic
→ spatio-temporal chaos

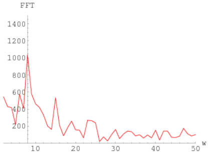
•interior crisis, intermittent saddle-node,
3-frequency?

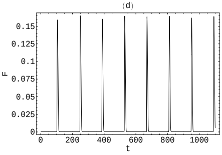
•Importance of hysteresis β

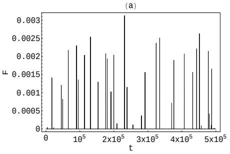
Bifurcation Diagram:magnetosphere?

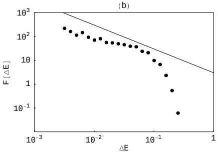
→Lower steady state → quiet state?
→Periodic solution → saw-tooth oscillations?
→Self-organization → turbulent self-similar dissipation?
→1st order phase transition → internal to directly driven?
→Upper steady state → steady magnetospheric convection?

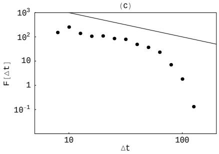




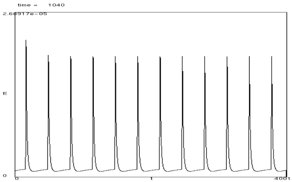


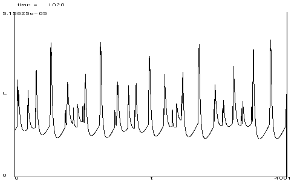






Bifurcation Diagram: spatio-temporal

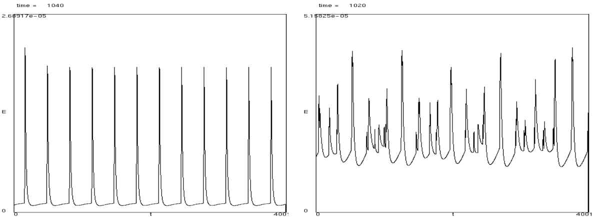




→Bifurcation to chaos is discontinuous

- $n = 3$ already chaotic
- Numerical Lyapunov exponent
- Bifurcation similar to Ruelle-Takens
- Quasiperiodic transitions
- Not a continuous Hoft bifurcation (Homoclinic?)

Aproximation 1D → with N



•Discontinuous bifurcation to chaos

→ $n = 3$ chaotic

→ Numerical Lyapunov exponent (3 frequency)

→ Bifurcation Ruelle-Takens

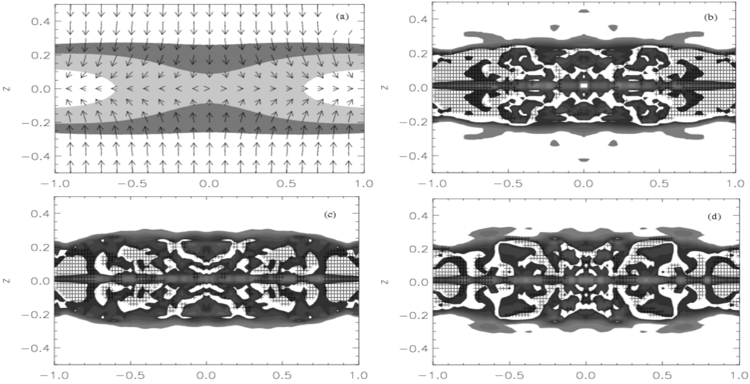
→ Quasiperiodic Transicions (saddle-node)

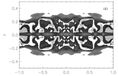
→ hysteresis

→ Deterministic vs. Stochastic

2D Model: self-consistent plasma

- 2-D $x - z$ plasma simulation: similar to SO





Model 2D → self-consistent

→Global state can be appreciated
→Intermittent dissipation and plasma flows (BBFS?)
→Complex dynamics

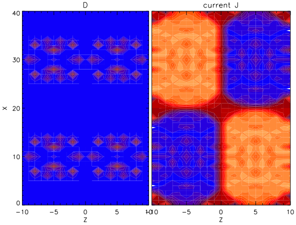
→Order and disorder

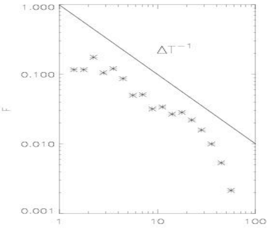
Model 2D → forced

- Model x-y with turbulent U (random stirring)
- fractional kinetics for fields

$$d_t A + U \cdot \nabla A = -\eta \nabla^2 A$$

$$U = \hat{y} \times \nabla \Psi \qquad \rightarrow \qquad \Psi = \sum_k \Psi_k e^{i(k \cdot r - \omega t) + i\phi_k}$$





- Force with Shell model?

Conclusions

→How to establish a global state
with underlying multifractality

→Hysteresis

→Spatio-temporal chaos

→Bifurcation diagram and transitions

- can we observe them?
- can we estimate parameters?
- Microphysics

Open Questions

→Energy cascade → Mode equation

$$\frac{dA_m}{dt} = -\sum_n \eta_{n-m} A_n \left(\frac{(2n+1)\pi}{2L}\right)^2 + S_m$$

- •Cascades with S_m ($m \neq 1$)
- •Deterministic vs. stochastic
- •Chaotic transitions in dimension and parameters
- •Multifractal Analysis:
  - → Singularity analysis (ξ_p, $P(\Delta X)$)
  - → Holder exponents

not ... The End