Standard Deviation Analysis (SDA)
versus
Diffusion Entropy Analysis (DEA)

UN/Austria Symposium on Data Analysis and Image Processing
for Space Applications and Sustainable Development:
Space Weather Data

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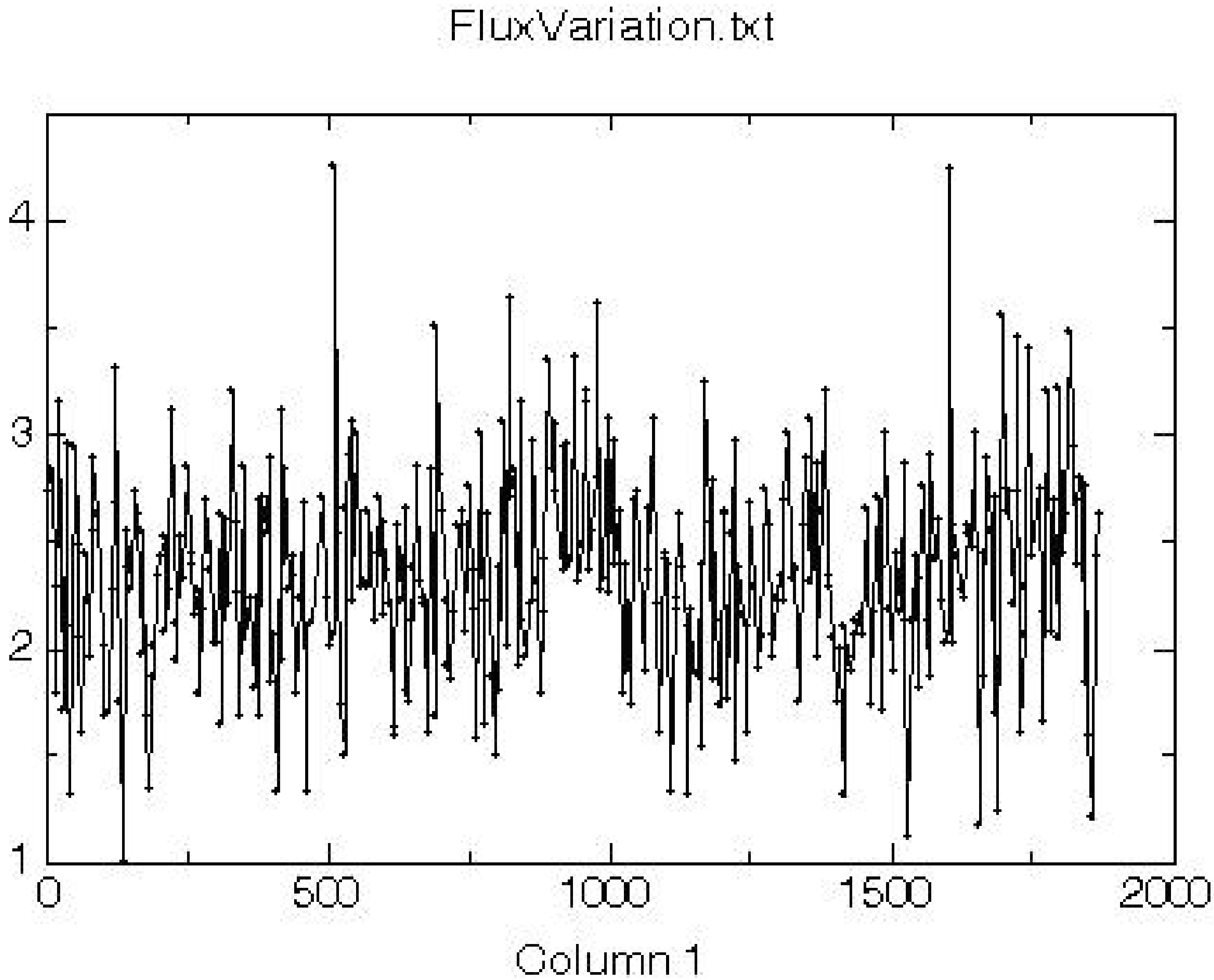
Time Series



Scaling Analysis of Time Series

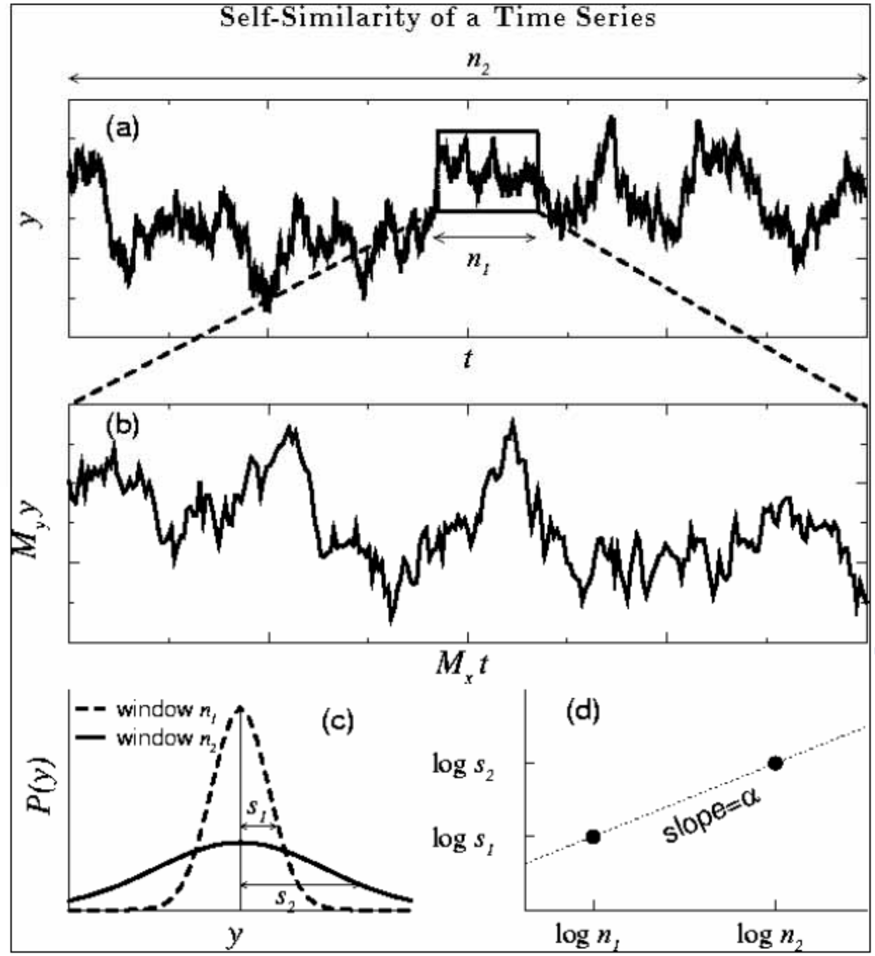
! Challenge in detecting and quantifying self-similar scaling of complex temporal processes.

Time series actually involves two different physical variable.

In mathematical terms: A time series is self-similar if:

$$y(t) \equiv a^{\alpha} y(\frac{t}{a})$$

α is called the scaling exponent.



Mapping Time Series to Diffusion Process

- By summing the terms of a time series we get a trajectory and the trajectory can be used to generate a diffusion process.

Let us consider a time series $\{\xi_i\}$ of N data: $\xi_1, \xi_2, \ldots\ldots, \xi_{N-1}, \xi_N$.
For any given time t, $1 \leq t \leq N$, we can find N-t+1 sub-sequences :

$$\xi_i^{(s)} \equiv \xi_{i+s} \qquad s = 0, 1, 2, \ldots\ldots, N-t$$

✓ For any of these sub-sequence we can build up diffusion trajectory, defined by the position:

$$x^{(s)}(t) = \sum_{i=1}^{t} \xi_i^{(s)} = \sum_{i=1}^{t} \xi_{i+s}$$

Fick's Laws and Brownian motion

$\vec{J} = -\underline{\underline{D}} \cdot \vec{\nabla} c$	1. Fick's Law
$\frac{dc}{dt} = -\underline{\underline{D}} \cdot \vec{\nabla}^2 c$	2. Fick's Law

- Einstein unified Fick's continuum formulation of diffusion with the Stochastic theory and obtained the probability distribution

$$p(x,t) = \frac{1}{\sqrt{4\pi D t}} \exp\left(-x^2 / 4Dt\right)$$

Gaussian distribution

Mean square displacement;

$$\left\langle x^2(t) \right\rangle \propto t$$

Anomalous Diffusion

Anomalous diffusion is characterised by

$$\left\langle x^2(t) \right\rangle \propto t^\gamma \qquad \text{with } \gamma \neq 1$$

Mandelbrot introduced a distribution to describe anomalous diffusion.

$$p(x,t) = \frac{1}{\sqrt{4\pi D t^\eta}} \exp\left(-x^2 \big/ 4Dt^\eta\right)$$

Second moment $<x^2(t)>=2Dt^\eta$

For $\eta=1$ the normal Brownian motion is recovered. The case $0<\eta<1$ corresponds to Sub diffusion and $\eta>1$ corresponds to Super diffusion.

Levy distribution

✓ Most of the Scaling analysis techniques study the scaling behavior of the second moment of the diffusion probability distribution.

✓ For some distributions the second and higher order moment diverge.

Paul Levy found a simple exception to the validity of the Central Limit Theorem.

Levy distribution is characterised by its Fourier transform:

$$f(k)=\exp(-a|k|^{\alpha}) \qquad 0<\alpha\leq 2$$

For asymptotic case, $f(x) \sim |x|^{-1-\alpha}$ as $|x| \rightarrow \infty$

How to find the Scaling behavior in this case!

Scaling Analysis of Time Series

- A diffusion process with scaling can be described by the probability density function (*pdf*)

$$p(x,t) = \frac{1}{t^{\delta}} F\left(\frac{x}{t^{\delta}}\right)$$

δ is called the *pdf* scaling exponent.

✓ The *Variance Scaling exponent* H of a diffusion process is defined by:

$$\Sigma^2(t) \sim t^{2H}$$

If $<x(t)>=0$, the variance, $\Sigma^2(t)=<x^2(t)>-<x(t)>^2$ coincides with the mean squared displacement.Then

$$\sum{}^2(t) = < x^2(t) > = \int_{-\infty}^{\infty} x^2 p(x,t)dx \sim t^{2H}$$

Scaling Analysis of Time Series

Now $p(x,t)=\frac{1}{t^{\delta}}F(\frac{x}{t^{\delta}})$

$$<x^2(t)>=\int_{-\infty}^{\infty}x^2\frac{1}{t^{\delta}}F(\frac{x}{t^{\delta}})dy$$

$$=t^{2\delta}\int_{-\infty}^{\infty}y^2F(y)dy$$ Where $y=x/t^{\delta}$

If $\int_{-\infty}^{\infty}y^2F(y)dy=$ Constant $<\infty$, $<x^2(t)>\sim t^{2\delta}$

➢ Thus the *pdf scaling exponent* δ and the *variance scaling exponent* H coincide in all cases with $\int y^2F(y)dy=\text{constant}<\infty$.

✓ This holds true, for example, in the normal Brownian motion.

Diffusion Entropy Analysis

- "Diffusion Entropy Analysis (DEA)" finds out the pdf scaling exponent δ.

- DEA is based on the evaluation of the Shannon entropy of the diffusion process.

Shannon entropy:

$$S(t) = -\int_{-\infty}^{\infty} p(x,t)\ln[p(x,t)]dx$$

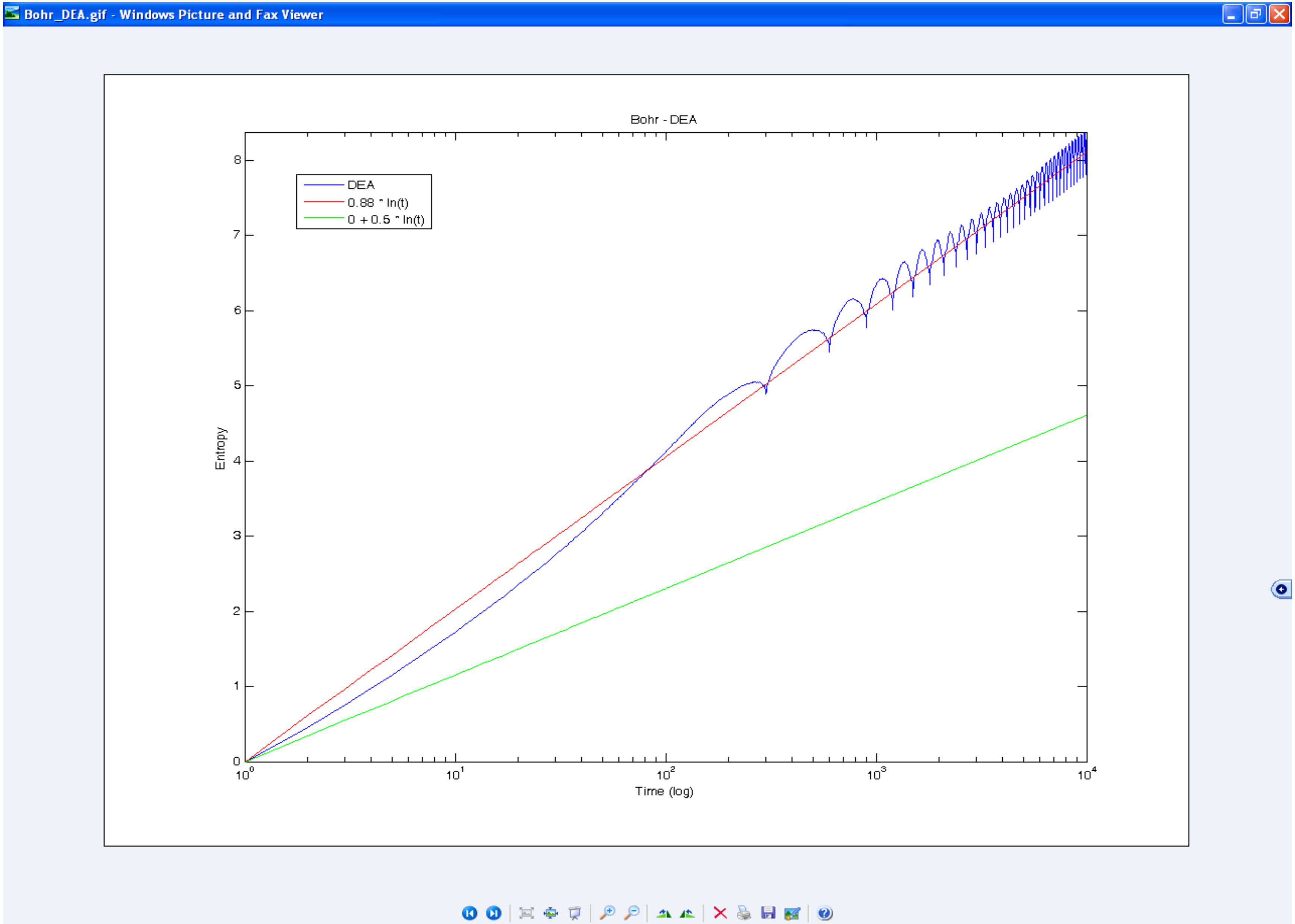
Using the scaling pdf we get, $S(t)=A+\delta\ln(t)$

✓ The slope of the log-linear plot of $S(t)$ against t gives the *pdf* scaling exponent δ.

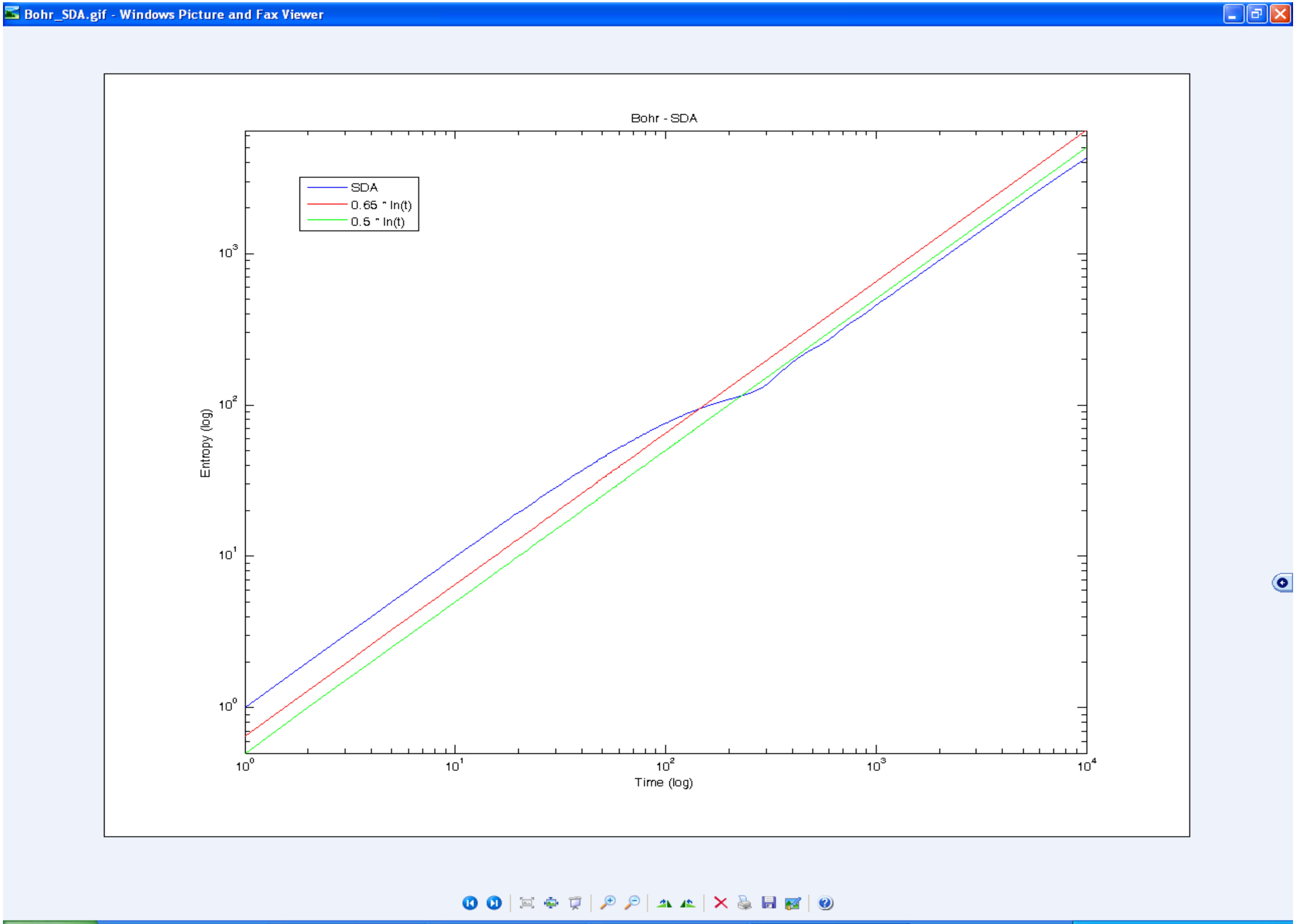
Relation between two scaling exponents

- $H = \delta = 0.5$: Normal diffusion
- $H = \delta \neq 0.5$: Fractional Brownian motion
- $H \neq \delta$: Levy Flight
- $\delta = 1/(3-2H)$: Levy Walk

DEA: delta=0.88



SDA: H=0.66


Bohr_SDA.gif - Windows Picture and Fax Viewer
Bohr - SDA
SDA
0.65 * ln(t)
0.5 * ln(t)
Entropy (log)
Time (log)
10^0
10^1
10^2
10^3
10^4