NONEXTENSIVE STATISTICAL MECHANICS:

OUTER SPACE AND WEATHER

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Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ **equiprobability**	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy ($q = 1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (q *real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

[C.T., J. Stat. Phys. **52**, 479 (1988)]

DEFINITIONS : *q – logarithm* : $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q – exponential : $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten :

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

TYPICAL SIMPLE SYSTEMS:

$$\text{e.g.,}\quad W(N) \propto \mu^N \quad (\mu > 1)$$

Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear/homogeneous Fokker-Planck equations, Gausssians

→ **Boltzmann-Gibbs entropy (additive)**

→ **Exponential dependences (Boltzmann-Gibbs weight, ...)**

TYPICAL COMPLEX SYSTEMS:

$$\text{e.g.,}\quad W(N) \propto N^\rho \quad (\rho > 0)$$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear/inhomogeneous Fokker-Planck equations, *q*-Gaussians

→ **Entropy S*q* (nonadditive)**

→ ***q*-exponential dependences (asymptotic power-laws)**

ADDITIVITY:

O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)\, S_q(A)\, S_q(B) \,,$$

S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \,,\ i.e.,\ S(N) \propto N \quad (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow S_{BG}(N) = k_B \ln W(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow S_q(N) = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow S_{q=1-1/\rho}(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\ 0 < \gamma < 1)$

$$\Rightarrow S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

$$\Rightarrow S_{\delta=1/\gamma}(N) \propto N \qquad \text{OK!}$$

IMPORTANT: $\quad \mu^N >> \nu^{N^\gamma} >> N^\rho \quad \text{if} \quad N >> 1$

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1 \quad \rightarrow \quad$ *Ising ferromagnet*

$0 < |\gamma| < 1 \quad \rightarrow \quad$ *anisotropic XY ferromagnet*

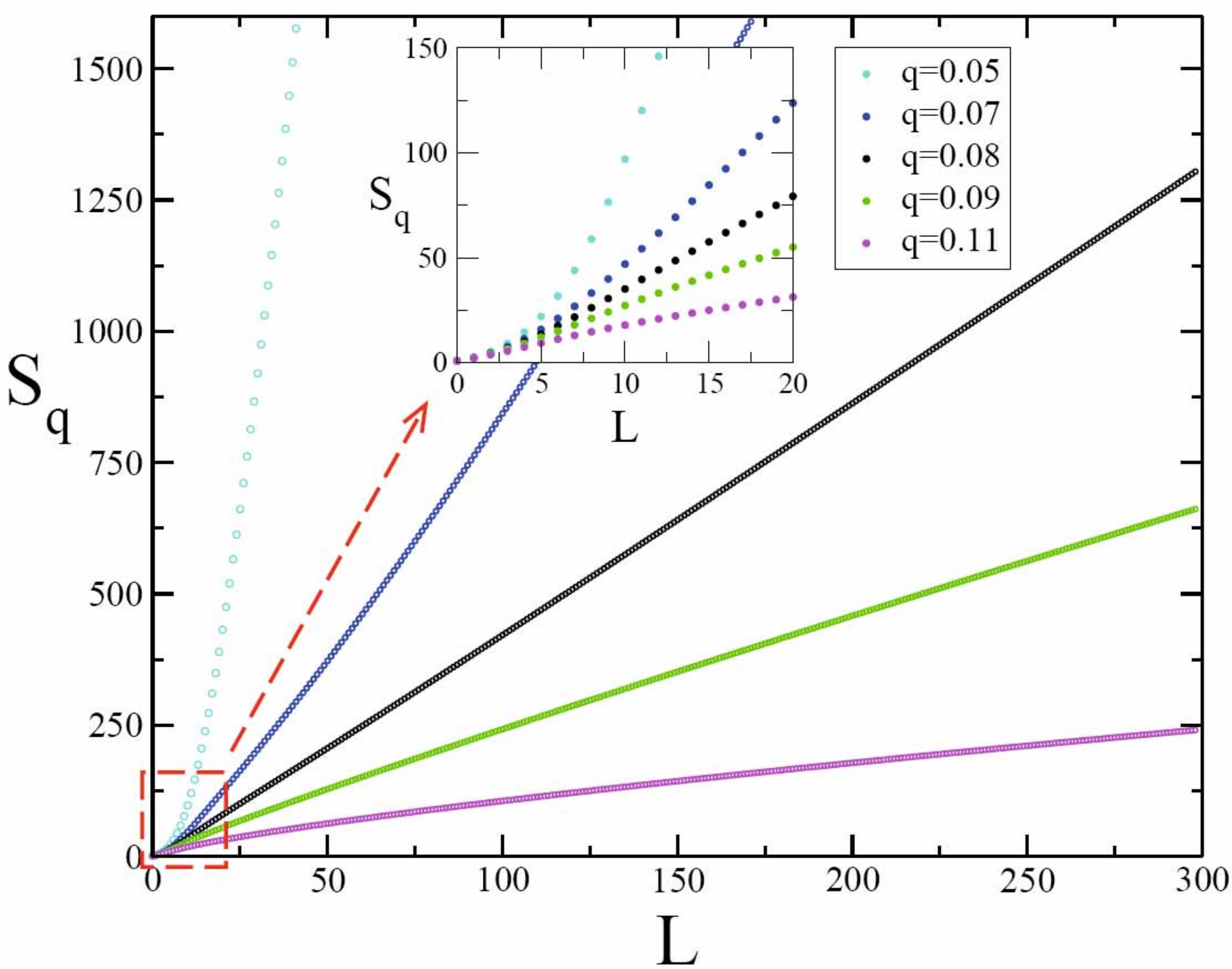
$\gamma = 0 \quad \rightarrow \quad$ *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \rightarrow \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

ISING MODEL



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

in P. Calabrese and J. Cardy, JSTAT P06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

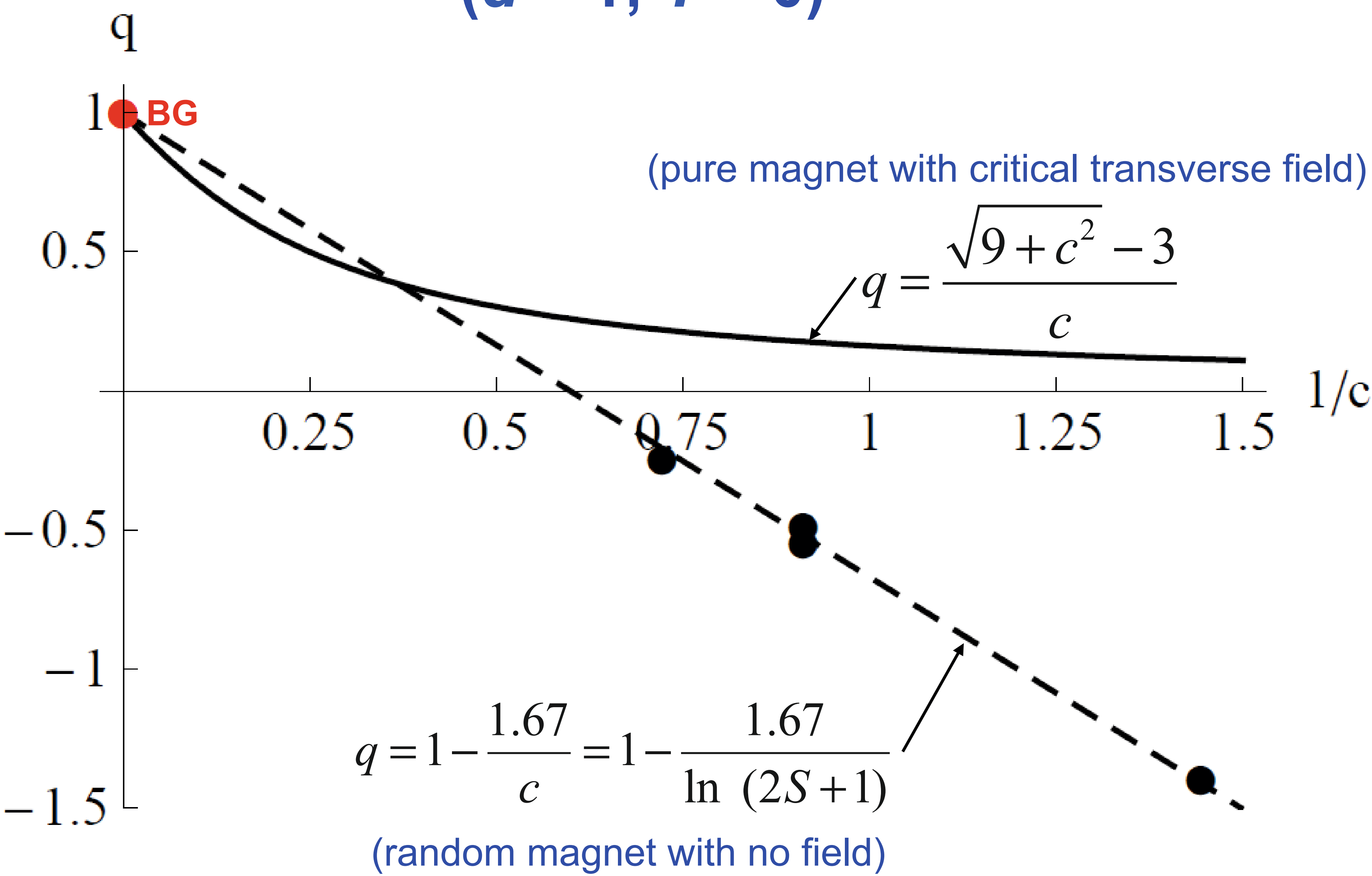
Ising and anisotropic XY ferromagnets $\Rightarrow \; c = \frac{1}{2} \;\Rightarrow\; q_{ent} = \sqrt{37}-6 \;\approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow \; c = 1 \;\Rightarrow\; q_{ent} = \sqrt{10}-3 \;\approx 0.1623$

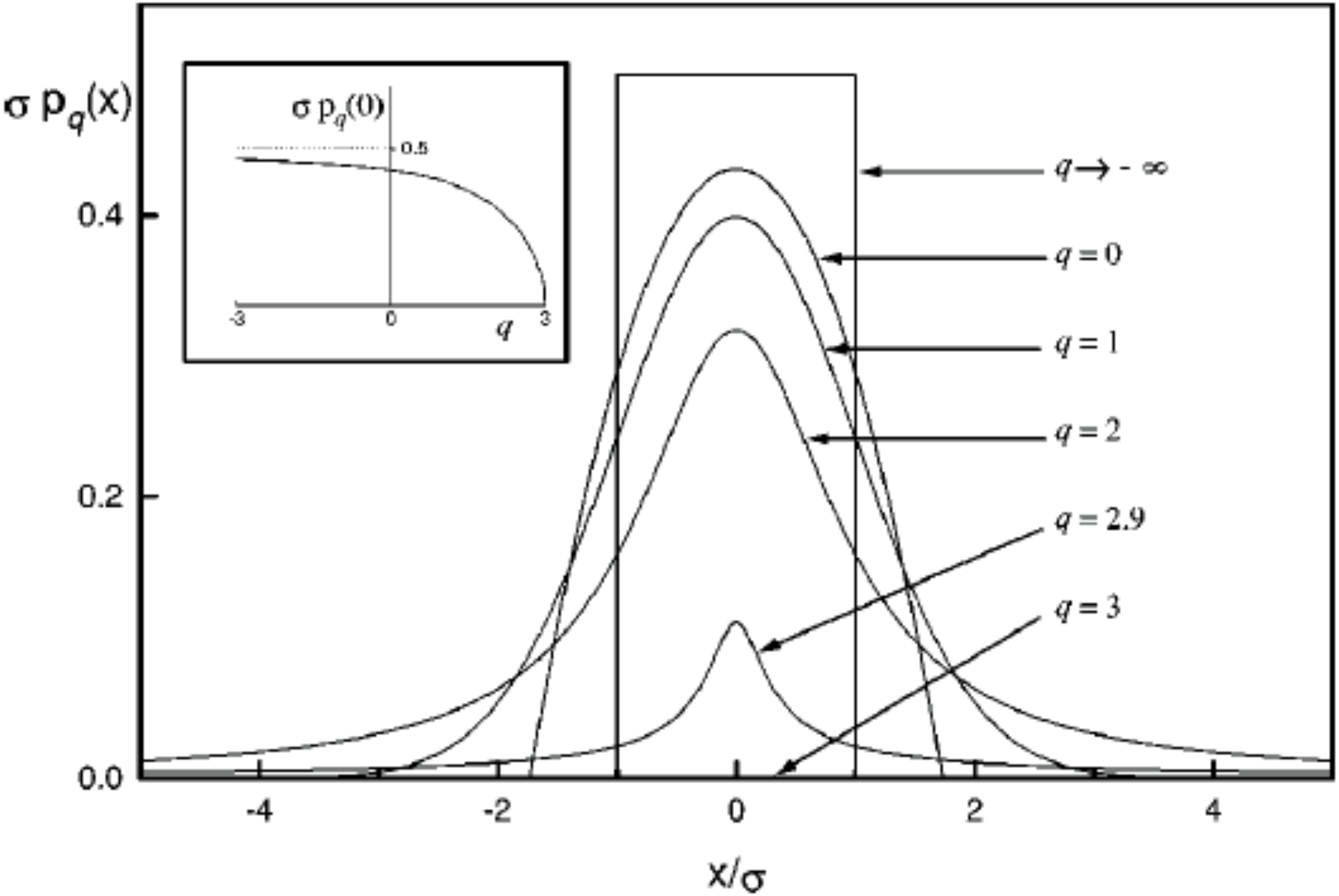
F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

$(d = 1;\ T = 0)$



A Saguia and MS Sarandy, Phys Lett A **374**, 3384 (2010)

q-GAUSSIANS: $p_q(x) \propto e_q^{-(x/\sigma)^2} \equiv \frac{1}{\left[1+(q-1)\,(x/\sigma)^2\right]^{\frac{1}{q-1}}}$ $(q<3)$



D. Prato and C. T., Phys Rev E **60**, 2398 (1999)

Milan j. math. 76 (2008), 307–328

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DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

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See also: H.J. Hilhorst, JSTAT P10023 (2010)

M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)

M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)

A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)

A. Plastino and M.C. Rocca (2013)

CENTRAL LIMIT THEOREM

$N^{1/[\alpha(2-q)]}$ - *scaled attractor* $F(x)$ *when summing* $N \to \infty$ q - *independent identical random variables*

with symmetric distribution $f(x)$ *with* $\sigma_Q \equiv \int dx\, x^2 [f(x)]^Q / \int dx\, [f(x)]^Q$ $\left(Q \equiv 2q-1,\ q_1 = \frac{1+q}{3-q} \right)$

	$q=1$ [*independent*]	$q \neq 1$ (*i.e.*, $Q \equiv 2q-1 \neq 1$) [*globally correlated*]
$\sigma_Q < \infty$ $(\alpha = 2)$	$F(x) =$ *Gaussian* $G(x)$, *with same* σ_1 *of* $f(x)$ Classic CLT	$F(x) = G_q(x) \equiv G_{(3q_1-1)/(1+q_1)}(x)$, *with same* σ_Q *of* $f(x)$ $G_q(x) \sim \begin{cases} G(x) & \text{if } \lvert x \rvert \ll x_c(q,2) \\ f(x) \sim C_q / \lvert x \rvert^{2/(q-1)} & \text{if } \lvert x \rvert \gg x_c(q,2) \end{cases}$ *with* $\lim_{q \to 1} x_c(q,2) = \infty$ S. Umarov, C. T. and S. Steinberg, Milan J Math 76, 307 (2008)
$\sigma_Q \to \infty$ $(0 < \alpha < 2)$	$F(x) =$ *Levy distribution* $L_\alpha(x)$, *with same* $\lvert x \rvert \to \infty$ *behavior* $L_\alpha(x) \sim \begin{cases} G(x) \\ \quad \text{if } \lvert x \rvert \ll x_c(1,\alpha) \\ f(x) \sim C_\alpha / \lvert x \rvert^{1+\alpha} \\ \quad \text{if } \lvert x \rvert \gg x_c(1,\alpha) \end{cases}$ *with* $\lim_{\alpha \to 2} x_c(1,\alpha) = \infty$ Levy-Gnedenko CLT	$F(x) = L_{q,\alpha}$, *with same* $\lvert x \rvert \to \infty$ *asymptotic behavior* $L_{q,\alpha} \sim \begin{cases} G_{\frac{2(1-q)-\alpha(1+q)}{2(1-q)-\alpha(3-q)},\, \alpha}(x) \sim C^*_{q,\alpha} / \lvert x \rvert^{\frac{2(1-q)-\alpha(3-q)}{2(1-q)}} \\ \text{(intermediate regime)} \\ G_{\frac{2\alpha q-\alpha+3}{\alpha+1},\, 2}(x) \sim C^L_{q,\alpha} / \lvert x \rvert^{(1+\alpha)/(1+\alpha q-\alpha)} \\ \text{(distant regime)} \end{cases}$ S. Umarov, C. T., M. Gell-Mann and S. Steinberg J Math Phys 51, 033502 (2010)

PHYSICAL REVIEW E **84**, 021121 (2011)

Group entropies, correlation laws, and zeta functions

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The notion of group entropy is proposed. It enables the unification and generaliztion of many different definitions of entropy known in the literature, such as those of Boltzmann-Gibbs, Tsallis, Abe, and Kaniadakis. Other entropic functionals are introduced, related to nontrivial correlation laws characterizing universality classes of systems out of equilibrium when the dynamics is weakly chaotic. The associated thermostatistics are discussed. The mathematical structure underlying our construction is that of formal group theory, which provides the general structure of the correlations among particles and dictates the associated entropic functionals. As an example of application, the role of group entropies in information theory is illustrated and generalizations of the Kullback-Leibler divergence are proposed. A new connection between statistical mechanics and zeta functions is established. In particular, Tsallis entropy is related to the classical Riemann zeta function.

$$S_q \leftrightarrow \frac{1}{(1-q)^{s-1}} \varsigma(s) \quad (q<1)$$

$$\text{with } \varsigma(s) \equiv \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}}$$

$$= \frac{1}{1-2^{-s}} \frac{1}{1-3^{-s}} \frac{1}{1-5^{-s}} \frac{1}{1-7^{-s}} \frac{1}{1-11^{-s}} \cdots$$

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TIME-EVOLVING STATISTICS OF CHAOTIC ORBITS OF CONSERVATIVE MAPS IN THE CONTEXT OF THE CENTRAL LIMIT THEOREM

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CONSERVATIVE MC MILLAN MAP:

$$x_{n+1} = y_n$$

$$y_{n+1} = -x_n + 2\mu \frac{y_n}{1 + y_n^2} + \varepsilon y_n$$

$\mu \neq 0 \Leftrightarrow$ nonlinear dynamics

G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)

$(\mu, \varepsilon) = (1.6, 1.2)$ $(\lambda_{max} \approx 0.05)$

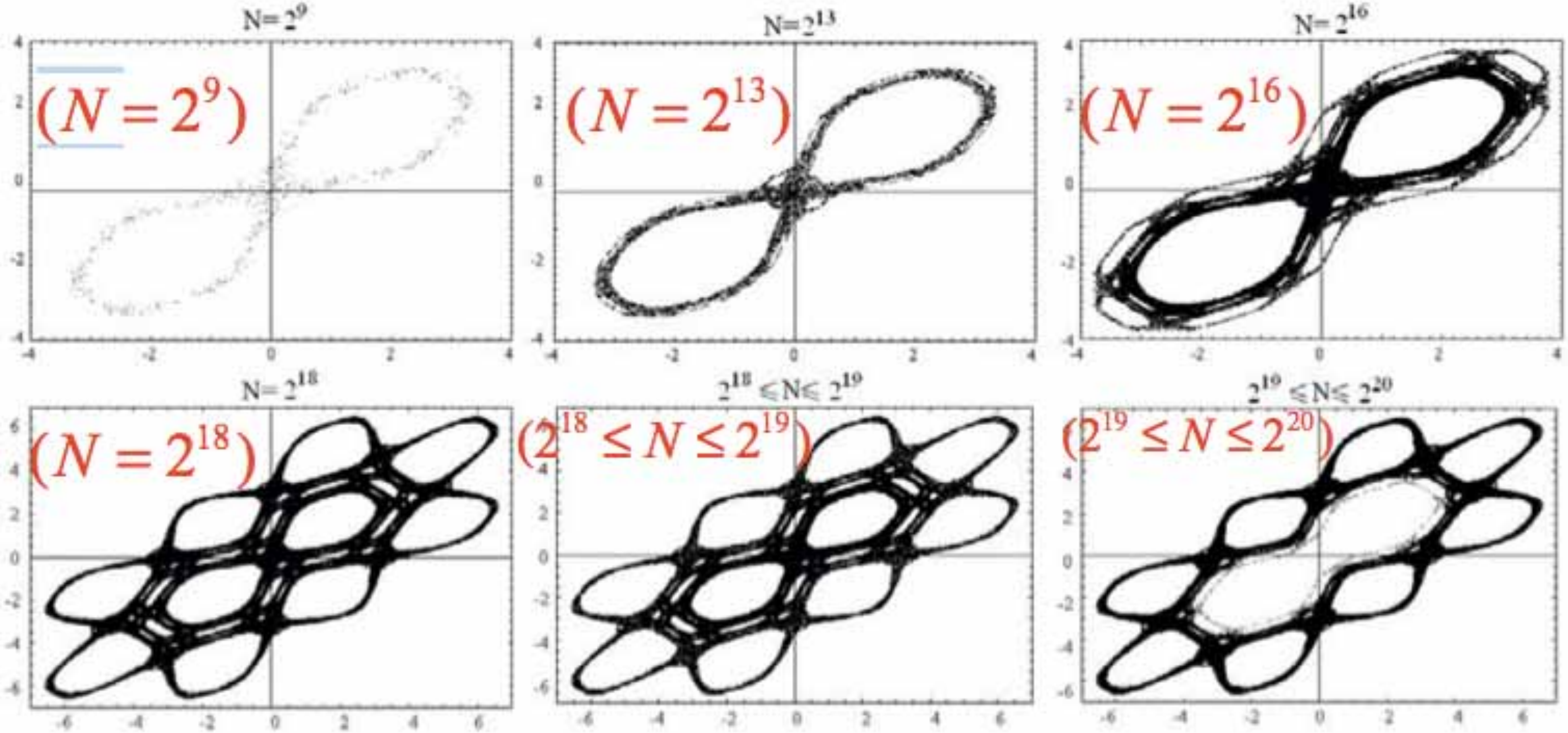
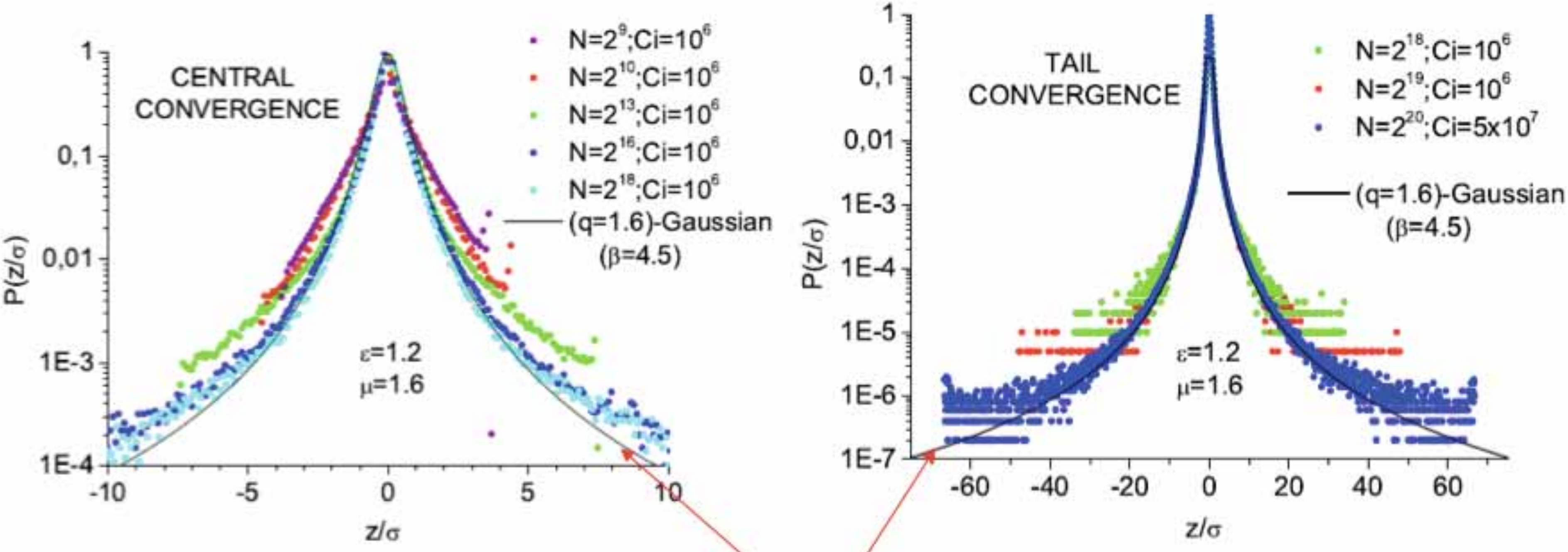


FIG. 10. Structure of phase space plot of Mc. Millan perturbed map for parameter values $\mu = 1.6$ and $\epsilon = 1.2$, starting form a randomly chosen initial condition in a square $(0, 10^{-6}) \times (0, 10^{-6})$, and for $i = 1 \ldots N$ $(N = 2^{10}, 2^{13}, N^{16}, N^{18})$ iterates.

G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)



$$p \propto e_q^{-\beta(z/\sigma)^2}$$

with $(q, \beta) = (1.6, 4.5)$

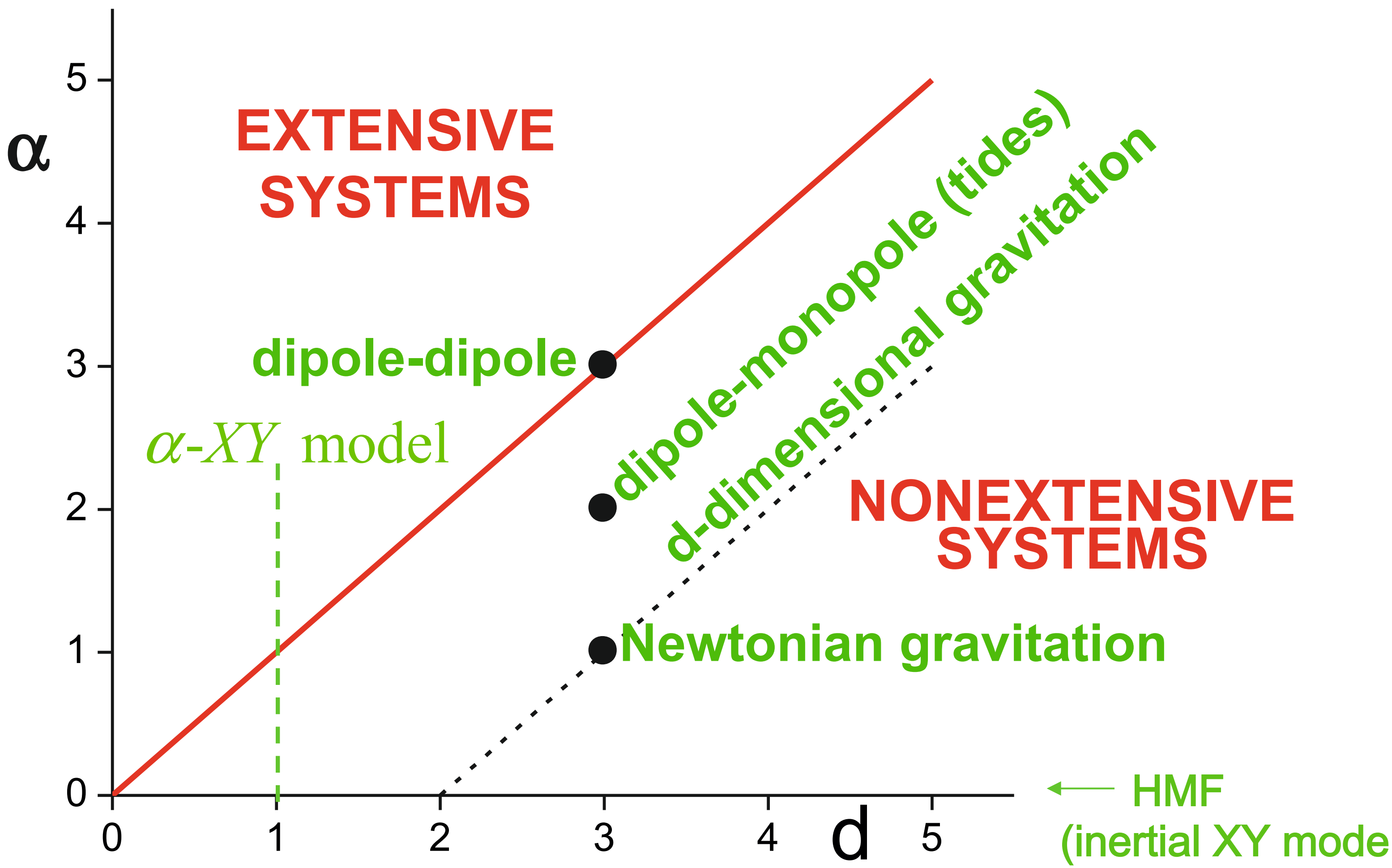
G. Ruiz, T. Bountis and C. T., Int J Bifurcat Chaos **22**, 1250208 (2012)

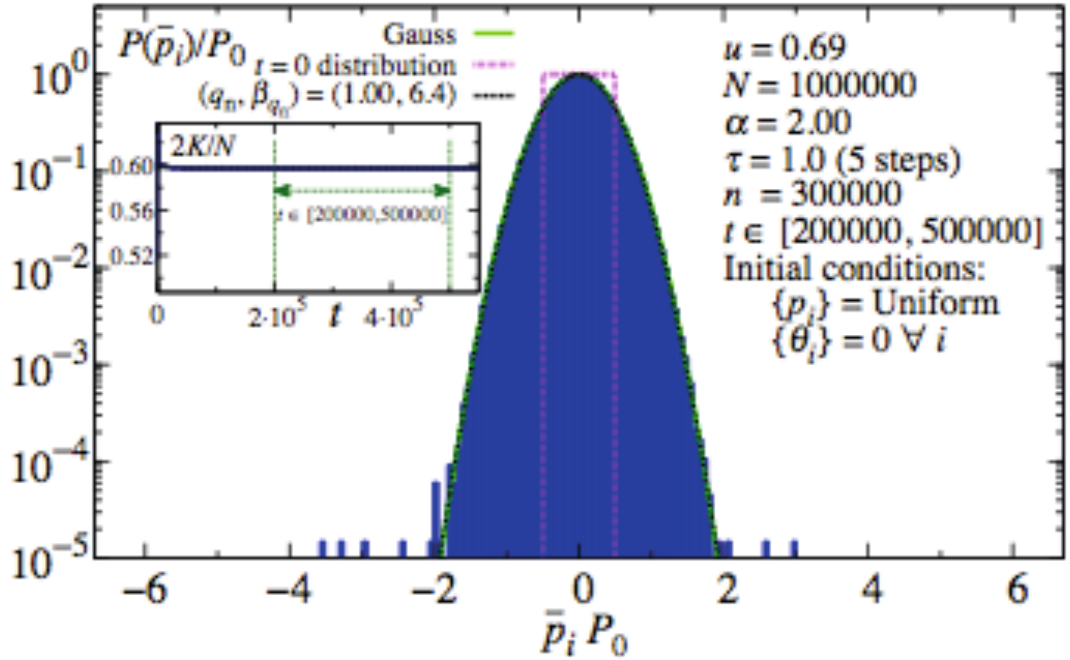
CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0,\ \ \alpha \geq 0)$$

integrable if $\alpha / d > 1$ *(short-ranged)*

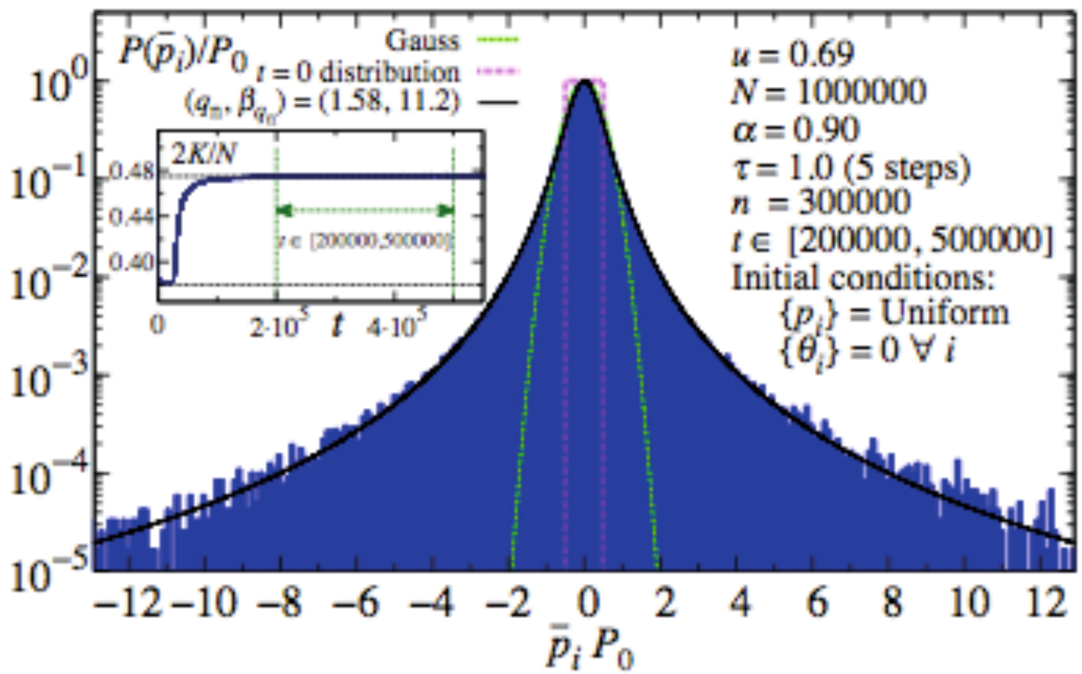
non-integrable if $0 \leq \alpha / d \leq 1$ *(long-ranged)*





$\alpha = 2$

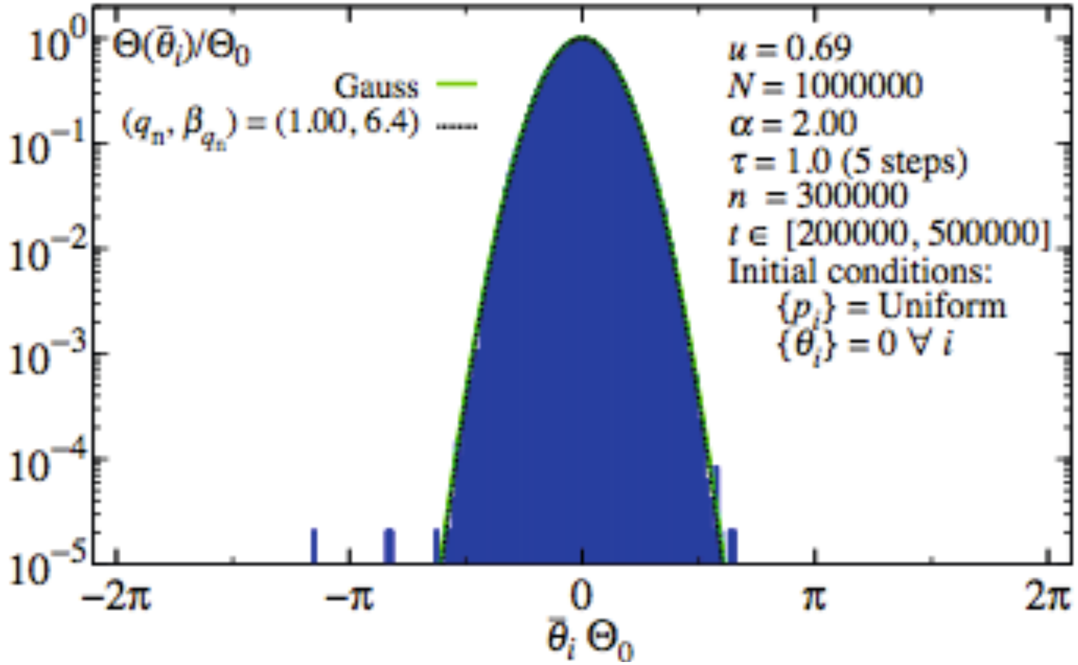
$q = 1$



$\alpha = 0.9$

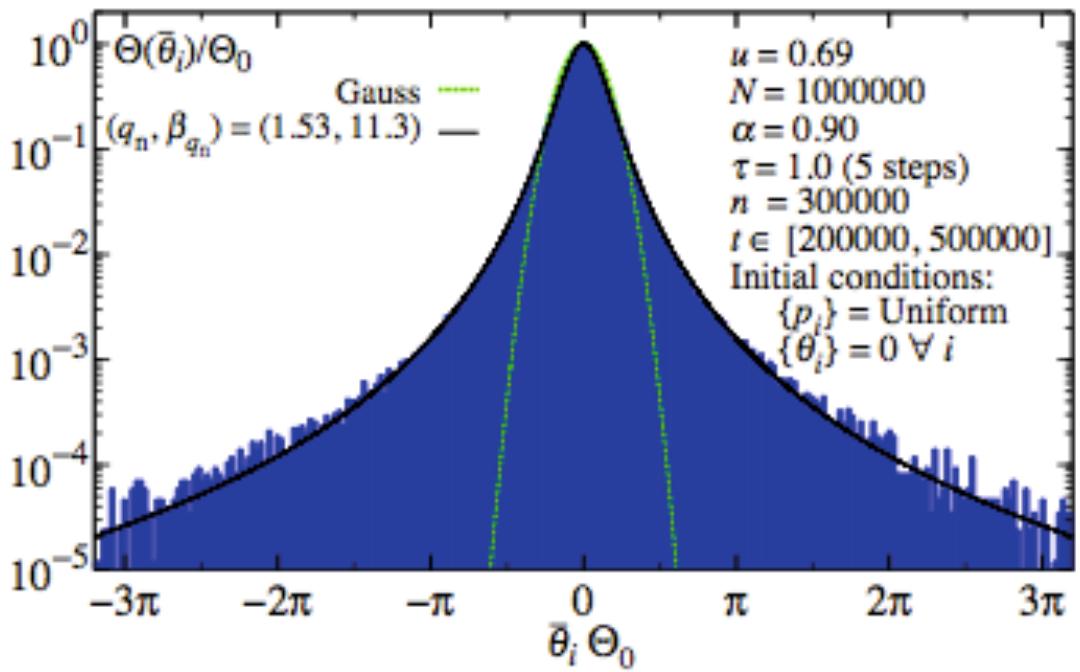
$q = 1.58$

L.J.L. Cirto, V.R.V. Assis and C. T., Pysica A (2013)



$\alpha = 2$

$q = 1$



$\alpha = 0.9$

$q = 1.53$

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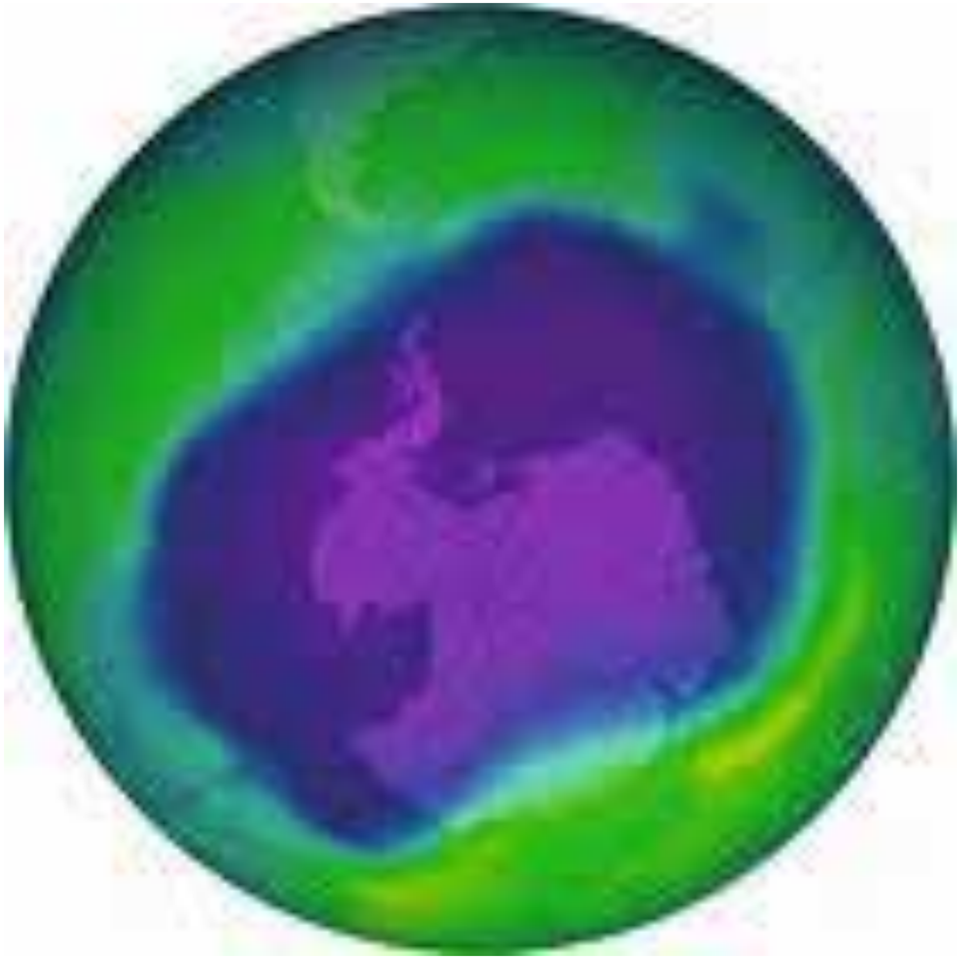
OZONE LAYER HOLE



10-50 Km above Earth

It absorbs 93-99% of the sun´s high frequency ultraviolet light

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Tsallis' *q*-triplet and the ozone layer

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ABSTRACT

Tsallis' *q*-triplet [C. Tsallis, Dynamical scenario for nonextensive statistical mechanics, Physica A 340 (2004) 1–10] is the best empirical quantifier of nonextensivity. Here we study it with reference to an experimental time-series related to the daily depth-values of the stratospheric ozone layer. Pertinent data are expressed in Dobson units and range from 1978 to 2005. After the evaluation of the three associated Tsallis' indices one concludes that nonextensivity is clearly a characteristic of the system under scrutiny.

Original data = mean value + long range tendency + annual oscillation + quasi-biannual one + Z_n

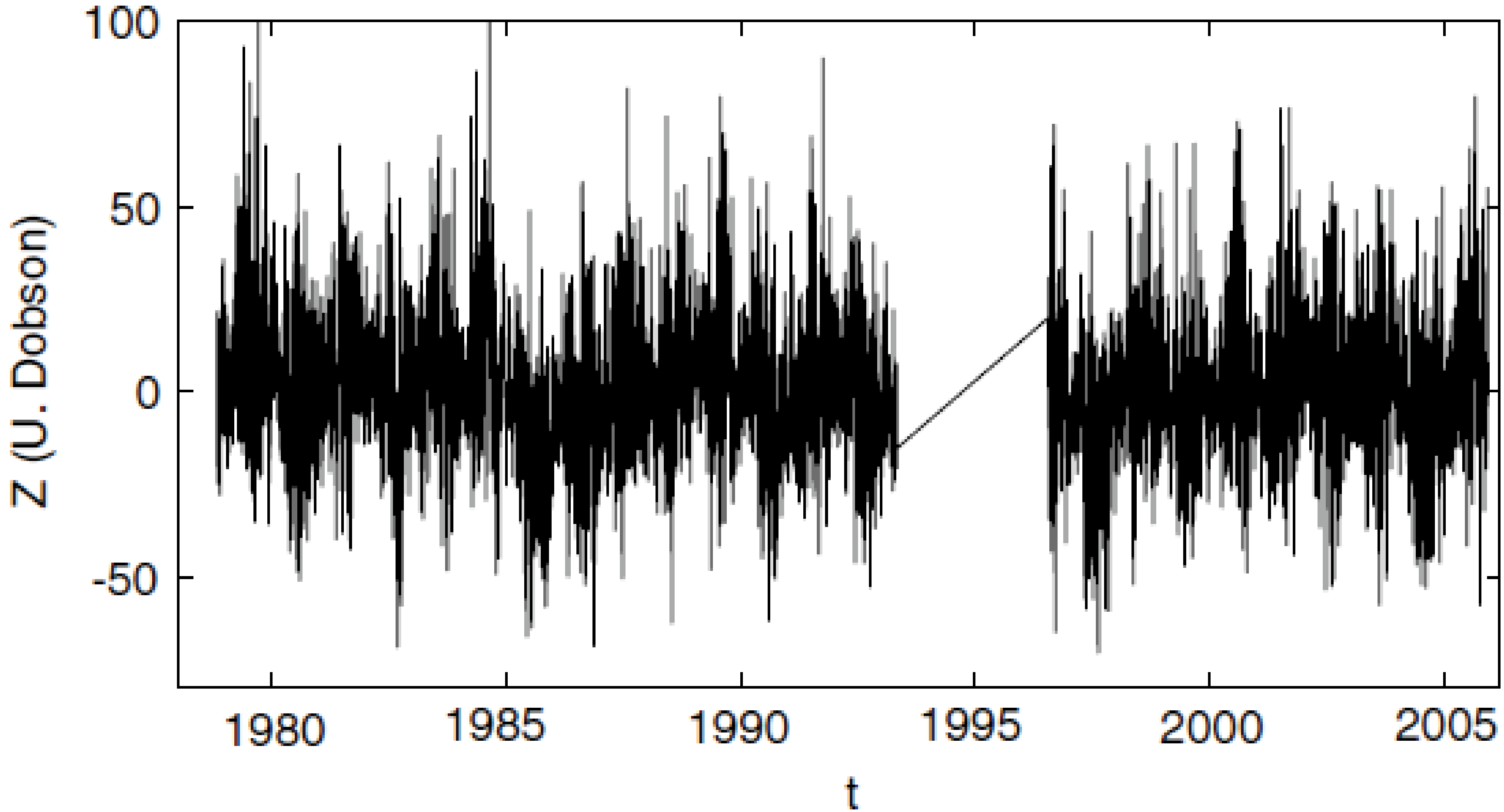

Fig. 1. Time-series Z_n. Daily values of the ozone layer over Buenos Aires city.

G.L. Ferri, M.F.R. Savio and A. Plastino, Physica A **389**, 1829 (2010)

$$q_{stat} = 1.32 \pm 0.06$$
$$q_{sens} = -8.1 \pm 0.02$$
$$q_{rel} = 1.89 \pm 0.02$$

hence

$$q_{sens} < 1 < q_{stat} < q_{rel}$$

G.L. Ferri, M.F.R. Savio and A. Plastino, Physica A **389**, 1829 (2010)

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Tsallis statistics and magnetospheric self-organization

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Superstorm

ABSTRACT

In this study we use Tsallis non-extensive statistics for a new understanding the magnetospheric dynamics and the magnetospheric self-organization during quiet and intensive superstorm periods. The q_{sens}, q_{stat}, and q_{rel} indices set known as the Tsallis q-triplet was estimated during both quiet and strongly active periods, as well as the correlation dimensions and Lyapunov exponents spectrum for magnetospheric bulk plasma flows data. The results obtained by our analysis clearly indicate the magnetospheric phase transition process from a high-dimensional quiet SOC state to a low-dimensional global chaotic state when superstorm events are developed. During such a phase transition process the non-extensive statistical character of the magnetospheric plasma is strengthened as the values of the q-triplet indices changes obtaining higher values than their values during the quiet periods.

Table 1
Summarize parameter values of magnetospheric dynamics: From the top to the bottom we show: changes of the ranges $\Delta\alpha$, $\Delta(D_q)$ of the multifractal profile. The q-triplet (q_sen, q_stat, q_rel) of Tsallis. The values of the maximum Lyapunov exponent (Li), the next Lyapunov exponent and the correlation dimension (D).

	Vx quiet	*Vx* storm
$\Delta\alpha = \alpha_{max} - \alpha_{min}$	1.069 ± 0.011	1.644 ± 0.03
$\Delta(D_q)$	0.721	1.205
q_sen	0.1343 ± 0.0267	0.3237 ± 0.0608
q_stat	1.120 ± 0.092	2.370 ± 0.056
q_rel	1.150 ± 0.080	2.910 ± 0.080
$L1$	≈ 0	>0
Li, ($i > 2$)	<0	<0
D (cor. Dim.)	>8	<4–5

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Nonextensivity in the solar magnetic activity during the increasing phase of solar cycle 23

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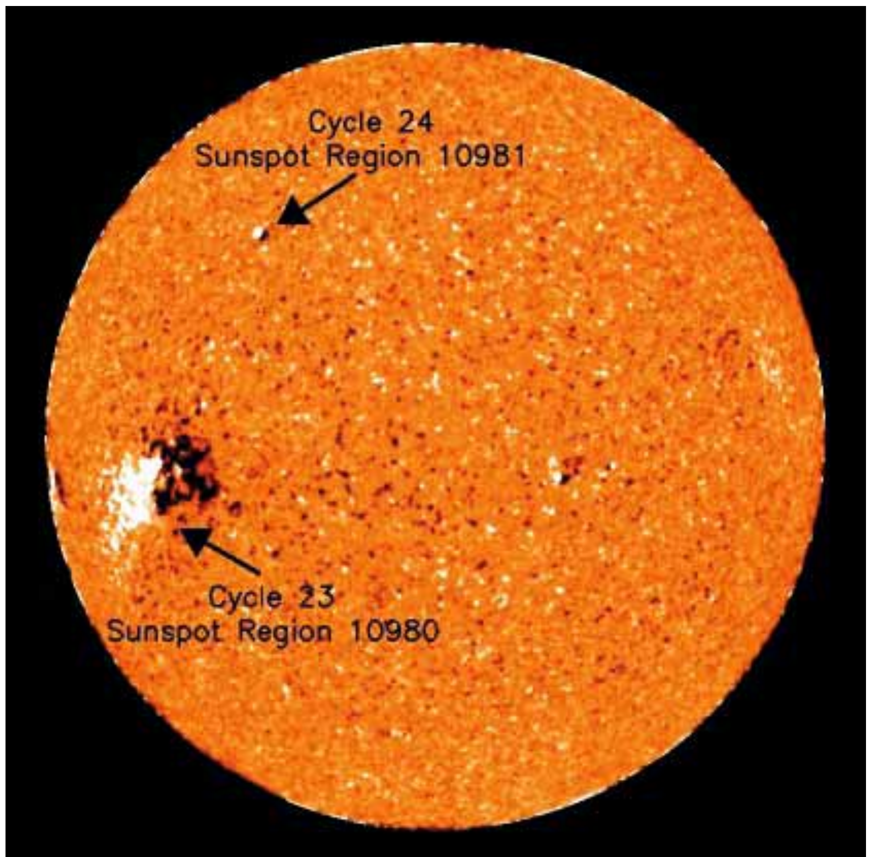
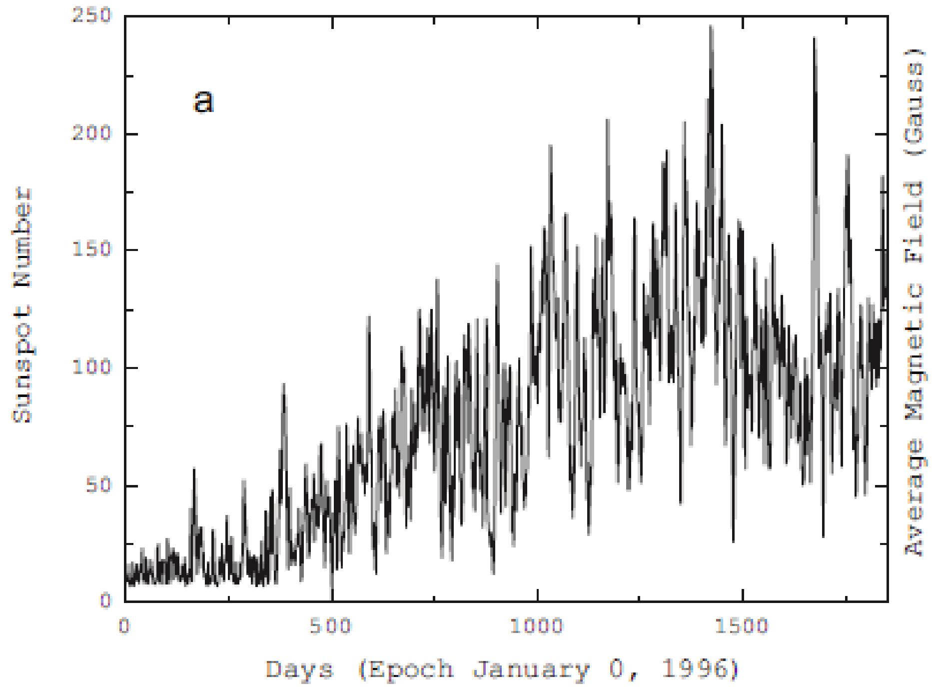
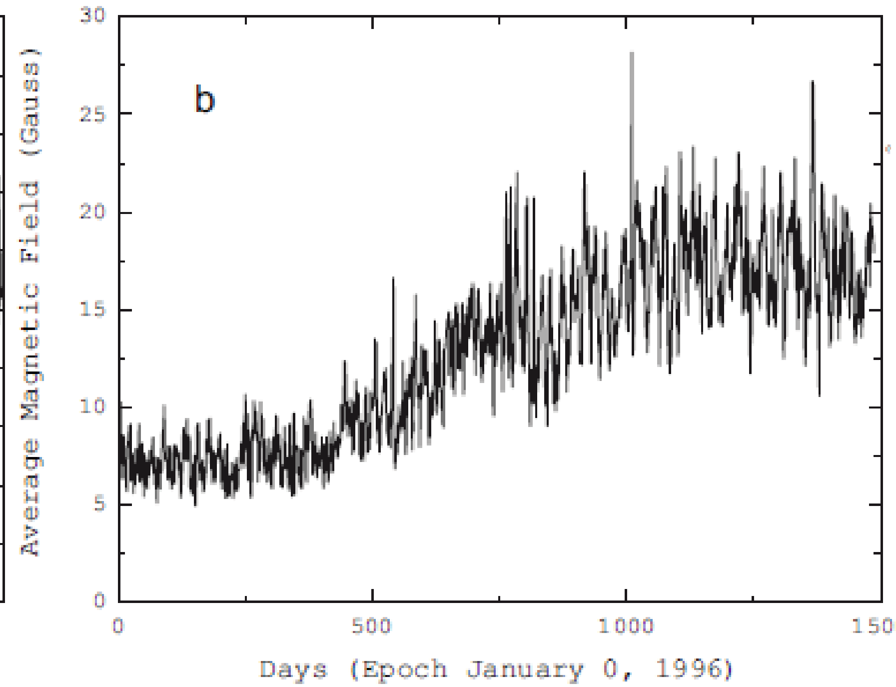
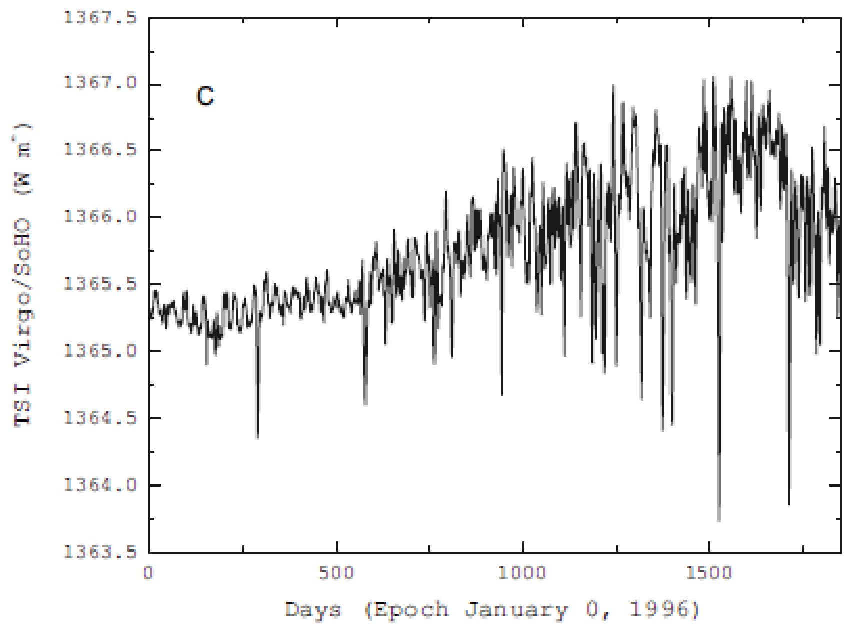




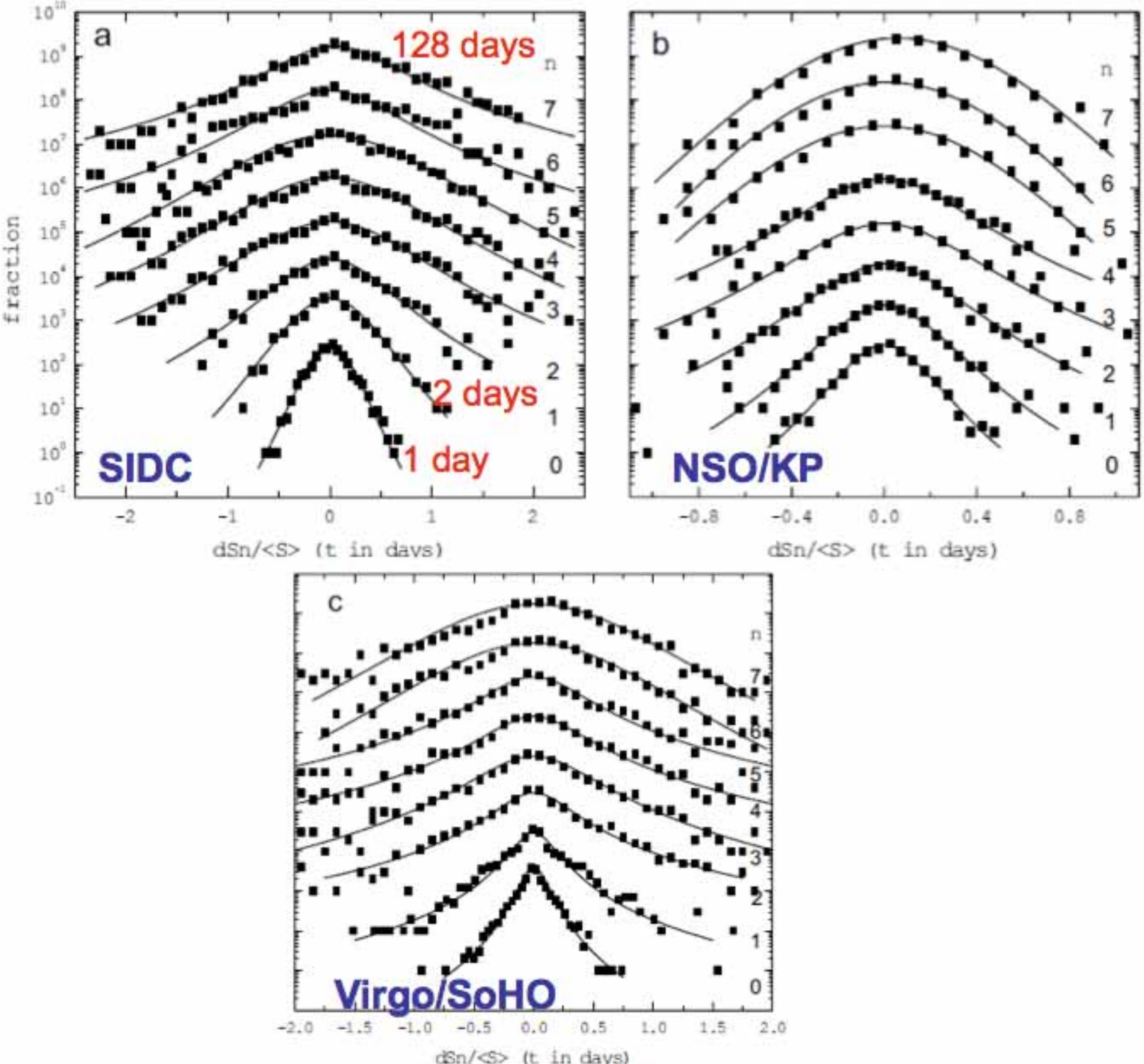
Sunspot number [Sunspot Index Data Center]



Magnetic field
[National Solar Observatory/Kitt Peak]



Total solar irradiance [Virgo/SoHO]


a
128 days
2 days
1 day
n
7
6
5
4
3
2
1
0
SIDC
fraction
-2
-1
0
1
2
dSn/<S> (t in days)
b
n
7
6
5
4
3
2
1
0
NSO/KP
-0.8
-0.4
0.0
0.4
0.8
dSn/<S> (t in days)
c
n
7
6
5
4
3
2
1
0
Virgo/SoHO
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
dSn/<S> (t in days)

$(\tau = 1 \text{ day})$

	q_{stat}	q_{sen}	q_{rel}
Solar Number [Sunspot Index Data Center]	1.31 ± 0.07	-0.71 ± 0.10	1
Magnetic Field [National Solar Observatory/Kitt Peak]	1.21 ± 0.06	-0.44 ± 0.07	1
Solar Total Irradiance [Virgo/SoHO]	1.54 ± 0.03	-0.52 ± 0.10	1

D.B. de Freitas and J.R. de Medeiros, Europhys Lett 88, 19001 (2010)

Physica A 391 (2012) 2154–2162

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Physica A

journal homepage: www.elsevier.com/locate/physa

Tsallis' statistics in the variability of El Niño/Southern Oscillation during the Holocene epoch

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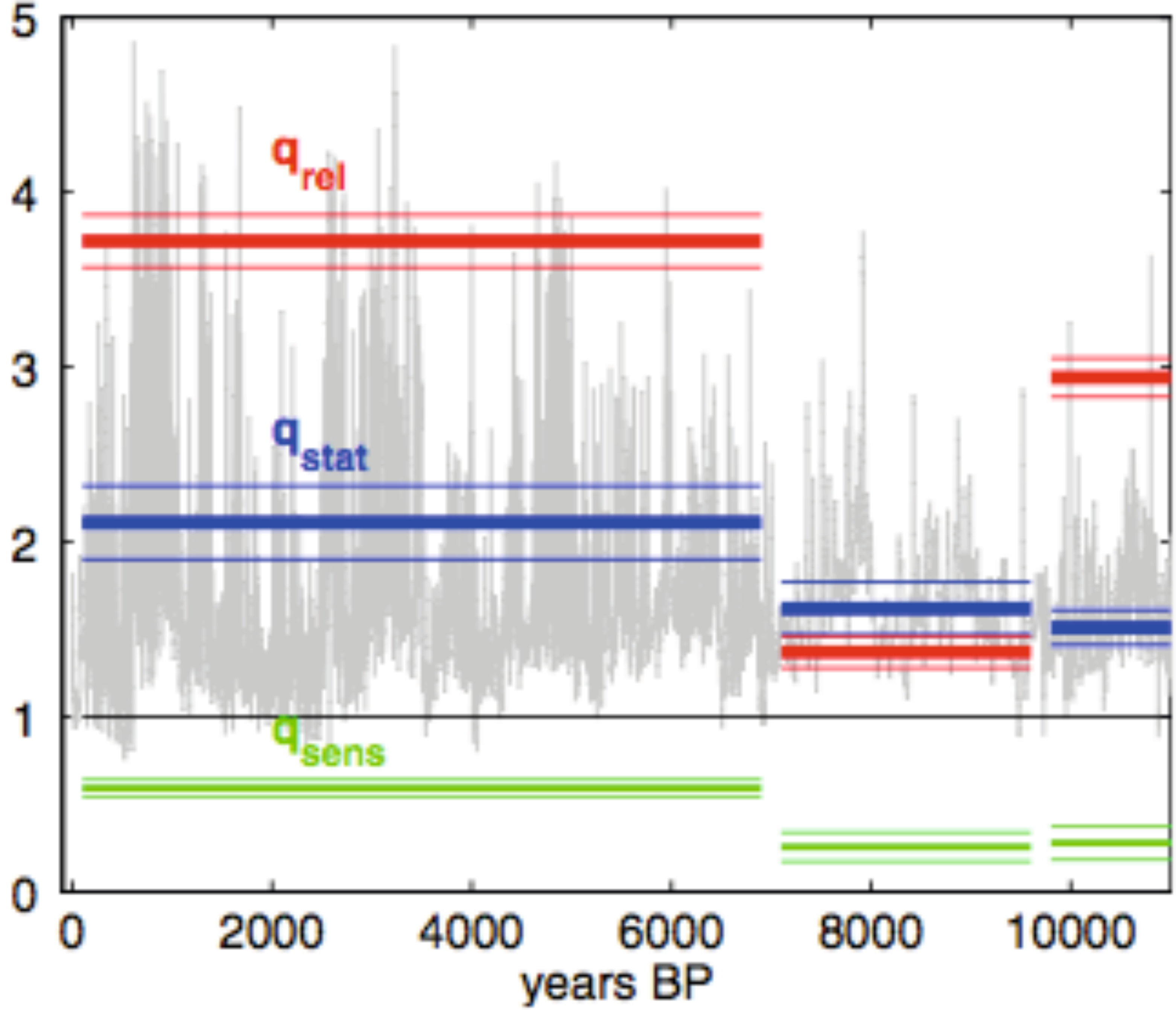

q_rel
q_stat
q_sens
5
4
3
2
1
0
0
2000
4000
6000
8000
10000
years BP

A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS

April 2013

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On the non-extensivity in Mars geological faults

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published online 3 May 2013

PACS 89.75.Da – Systems obeying scaling laws
PACS 89.75.-k – Complex systems
PACS 96.30.Gc – Mars

Abstract – A non-extensive statistical physics approach is tested for the first time in a planetary scale, for the fault length distribution in Mars estimated a non-extensive q-parameter equal to 1.277 for normal faults and 1.114 for thrust ones. The latter support the conclusion that the fault systems in Mars are subadditive ones in agreement with recent observations for faults in Earth and Valles Marineris extensional province, Mars. In addition, an analysis of the global Mars fault system as a mixed one, consisted of the normal and thrust subsystems with different q-parameters is presented, leading to $q = 1.22$.

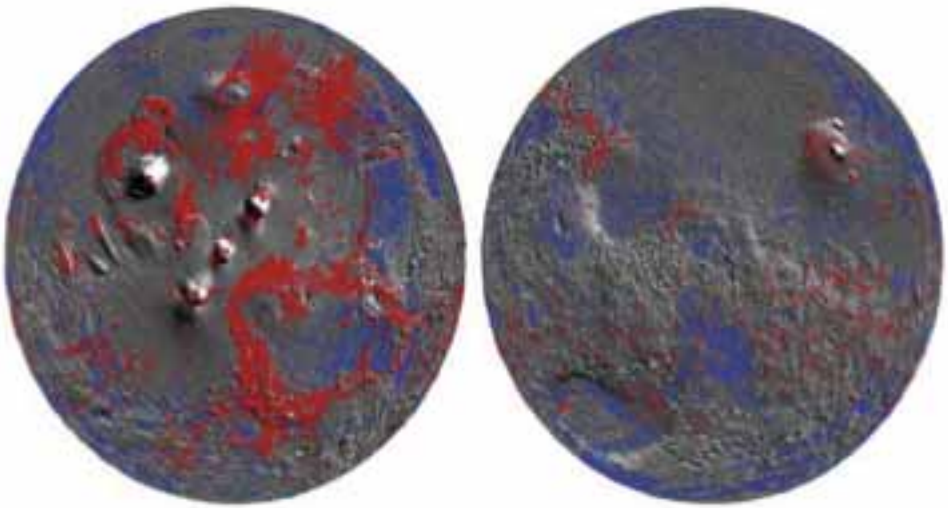
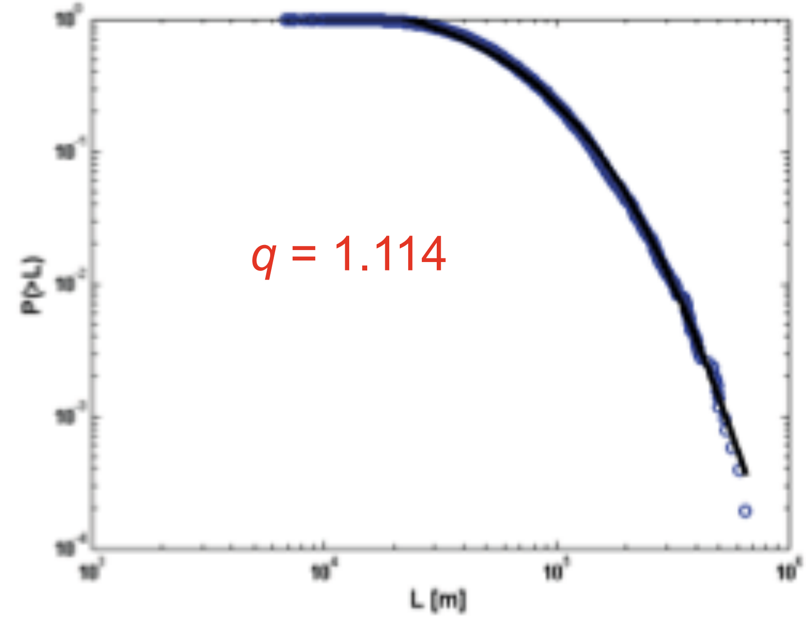
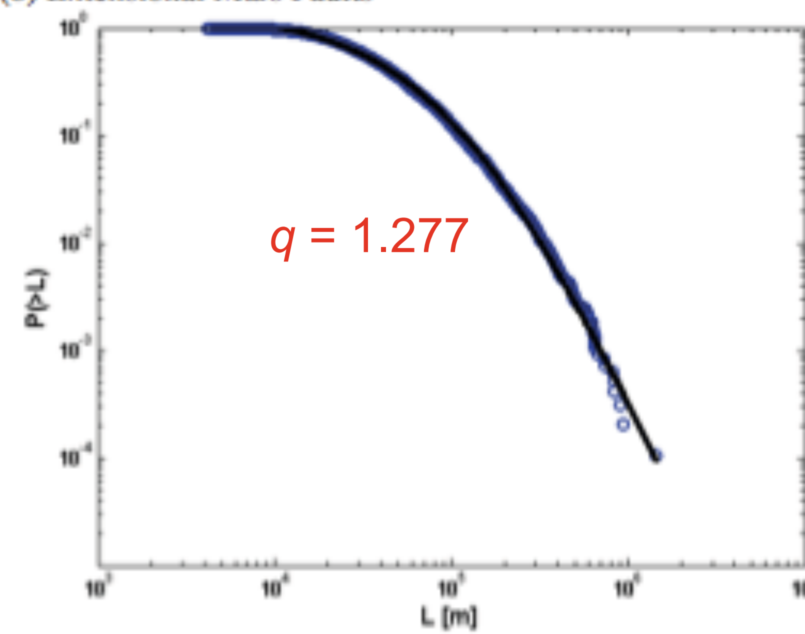


Fig. 1: (Colour on-line) Global distribution of faults on Mars Western hemisphere (left) and eastern hemisphere (right), extracted from [22] and [42]. The extensional faults (in red) are mainly concentrated in the Western hemisphere, while the contractional faults, are located in both Mars hemispheres.

(a) Compressional Mars faults



(b) Extensional Mars Faults



A&A 539, A158 (2012)
DOI: 10.1051/0004-6361/201117767

Astronomy & Astrophysics

Nonextensive distributions of asteroid rotation periods and diameters

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ABSTRACT

Context. We investigate the distribution of asteroid rotation periods from different regions of the solar system and diameter distributions of near-Earth asteroids (NEAs).
Aims. We aim to verify if nonextensive statistics satisfactorily describes the data.
Methods. Light curve data were taken from the Planetary Database System (PDS) with Rel ≥ 2. We also considered the taxonomic class and region of the solar system. Data of NEA were taken from the Minor Planet Center.
Results. The rotation periods of asteroids follow a q-Gaussian with $q = 2.6$ regardless of taxonomy, diameter, or region of the solar system of the object. The distribution of rotation periods is influenced by observational bias. The diameters of NEAs are described by a q-exponential with $q = 1.3$. According to this distribution, there are expected to be 994 ± 30 NEAs with diameters greater than 1 km.

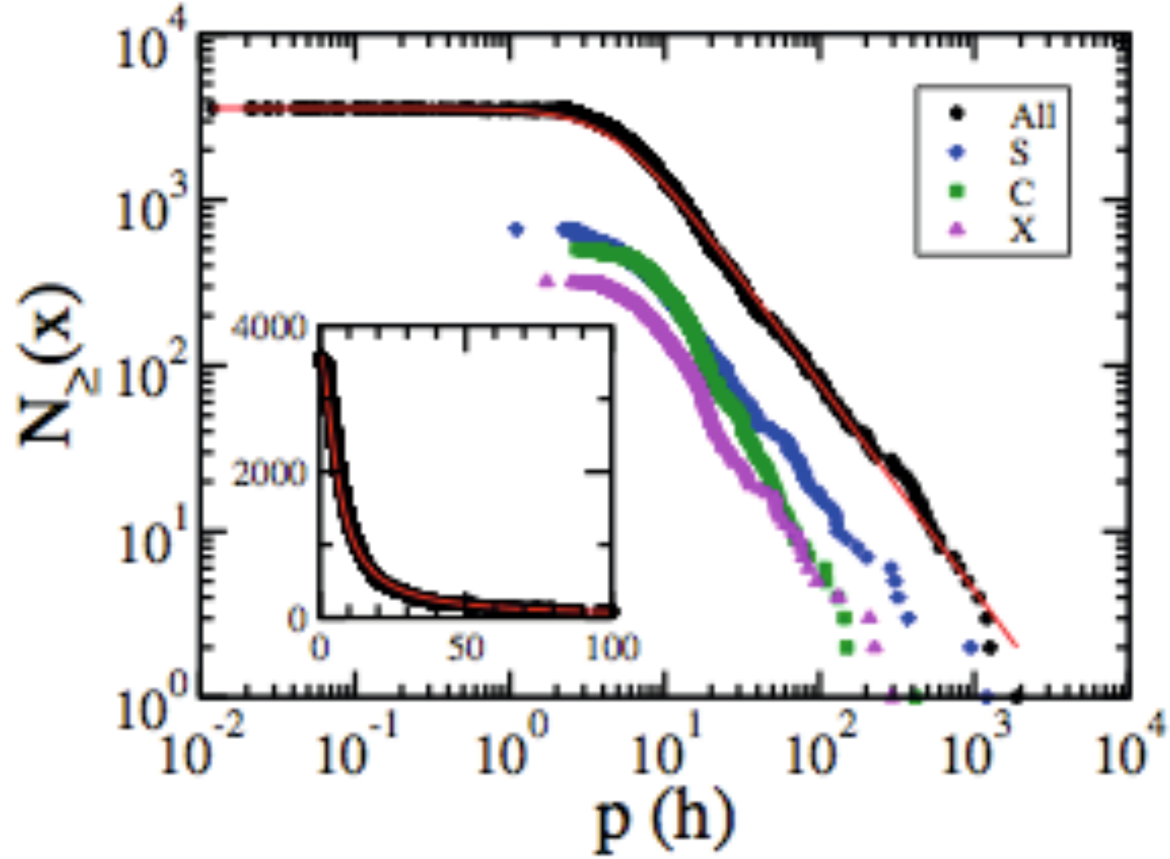


Fig. 3. Log-log plot of the decreasing cumulative distribution of periods of 3567 asteroids (dots) with Rel $\geq$ 2 taken from the PDS (NASA) and a q-Gaussian distribution ($N_{\geq}(p) = M \exp_q(-\beta_q p^2)$) (solid line), with $q = 2.6$, $\beta_q = 0.025$ h^{-2}, $M = 3567$. The other curves are 663 S-complex asteroids (diamonds, blue online), 503 C-complex asteroids (squares, green online), 321 X-complex asteroids (triangles, magenta online). Inset shows the 3567 asteroids and the q-Gaussian in a linear-linear plot.

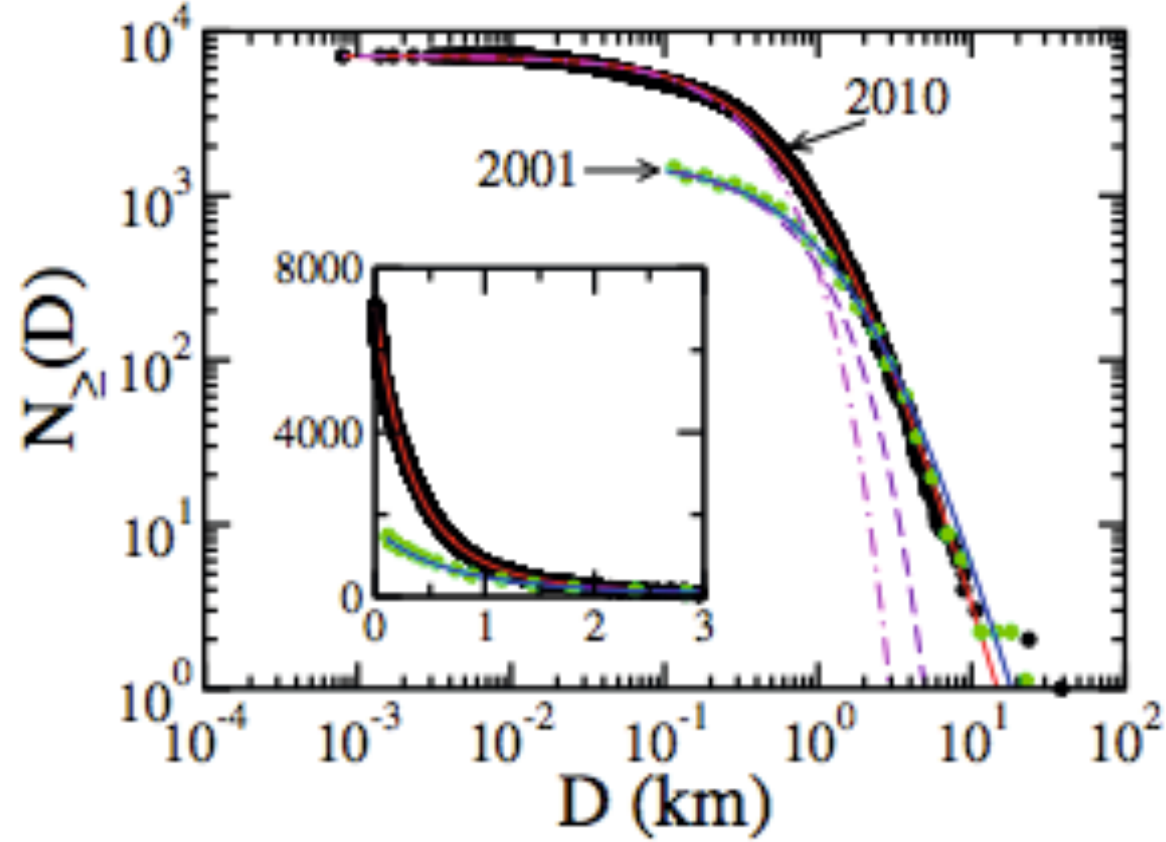


Fig. 4. Decreasing cumulative distribution of diameters of known NEAs in 2001 (1649 objects, green dots) and in 2010 (7078 objects, black dots). Solid lines are best fits of q-exponentials ($N_{\geq}(D) = M \exp_q(-\beta_q D)$). Blue line (2001): $q = 1.3$, $\beta_q = 1.5$ km^{-1}, $M = 1649$, red line (2010): $q = 1.3$, $\beta_q = 3$ km^{-1}, $M = 7078$. Normal exponentials ($q = 1$) are displayed in the main panel for comparison (dashed violet, with $\beta_1 = 1.5$ km^{-1}, $M = 1649$, and dot-dashed magenta, with $\beta_1 = 3$ km^{-1}, $M = 7078$).

LHC (Large Hadron Collider)

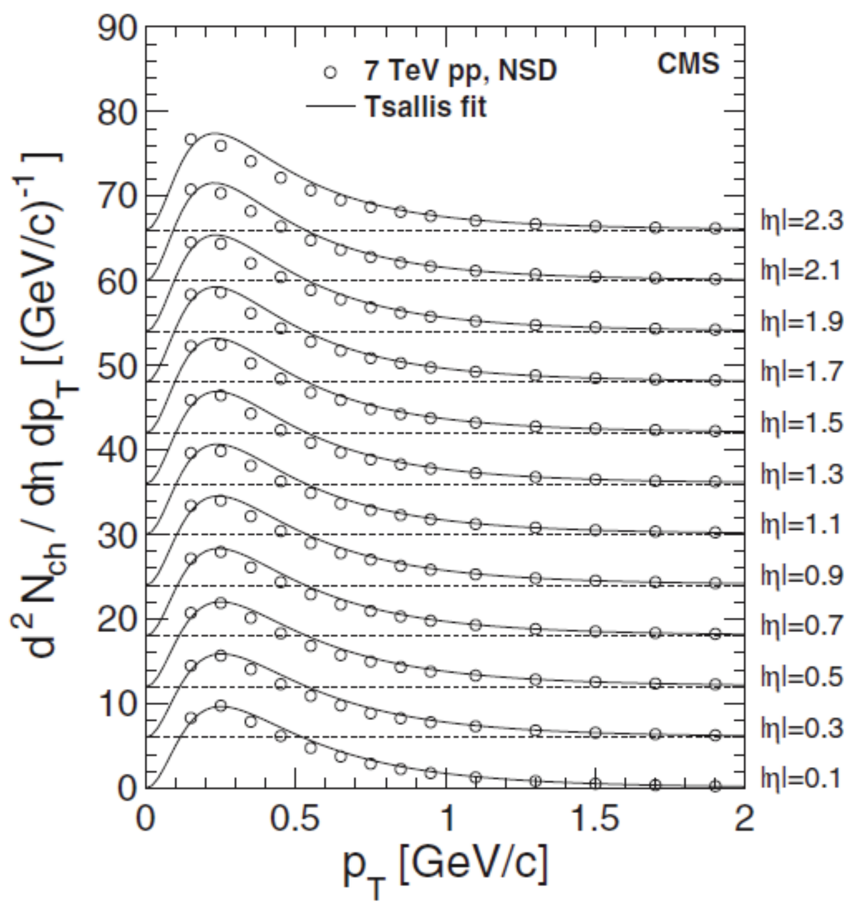
CMS (Compact Muon Solenoid) detector

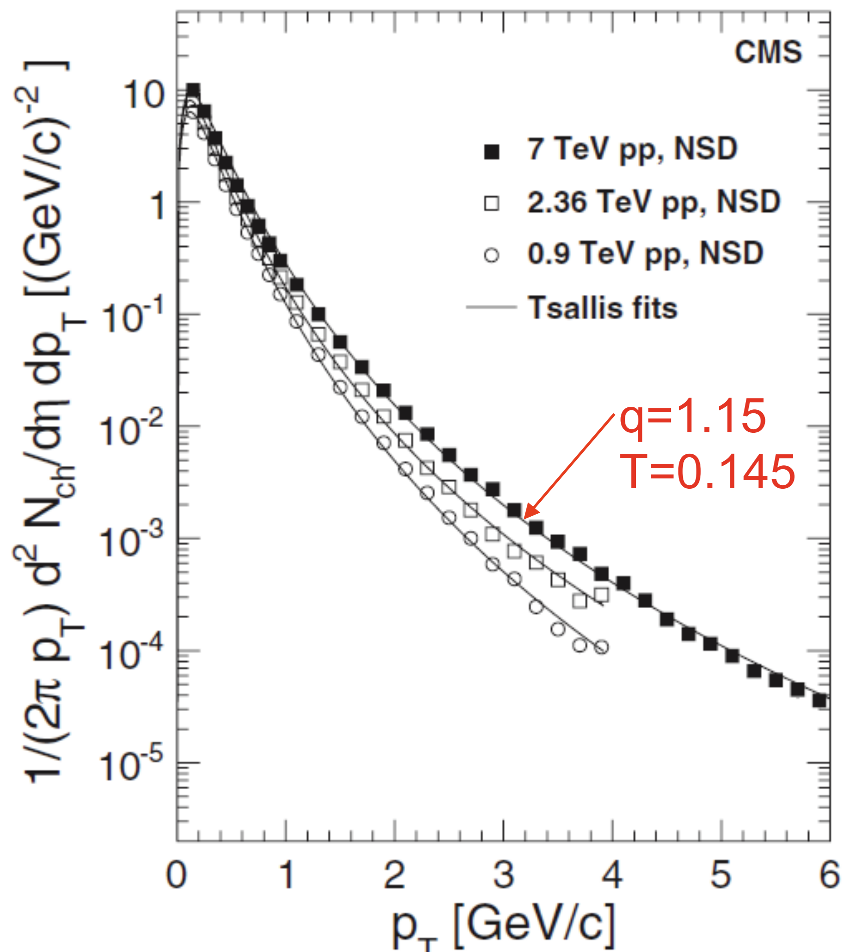
~ 2500 scientists/engineers from 183 institutions of 38 countries

Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in pp Collisions at $\sqrt{s} = 7$ TeV

V. Khachatryan *et al.**
(CMS Collaboration)

(Received 18 May 2010; published 6 July 2010)



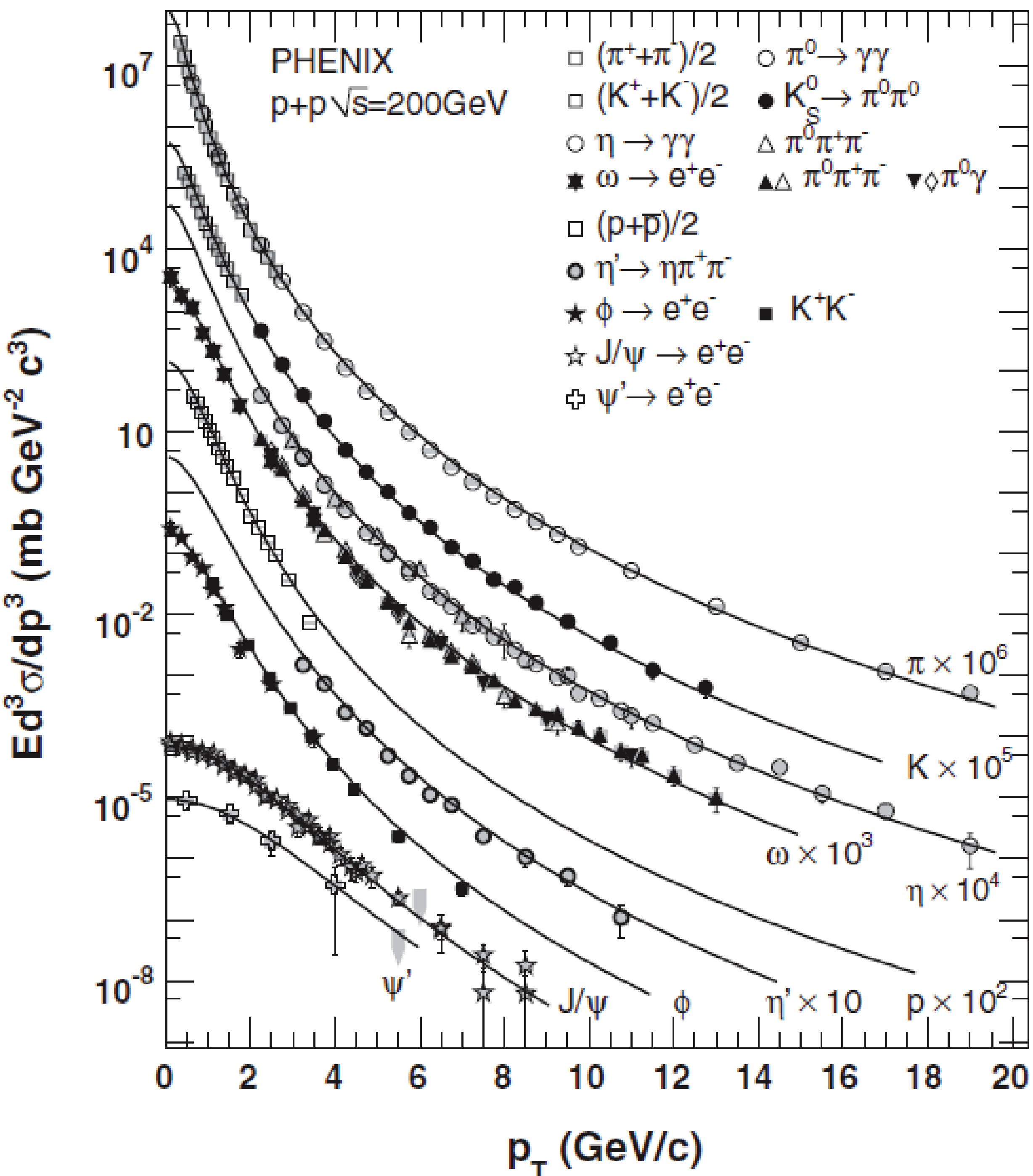


PHYSICAL REVIEW D **83,** 052004 (2011)

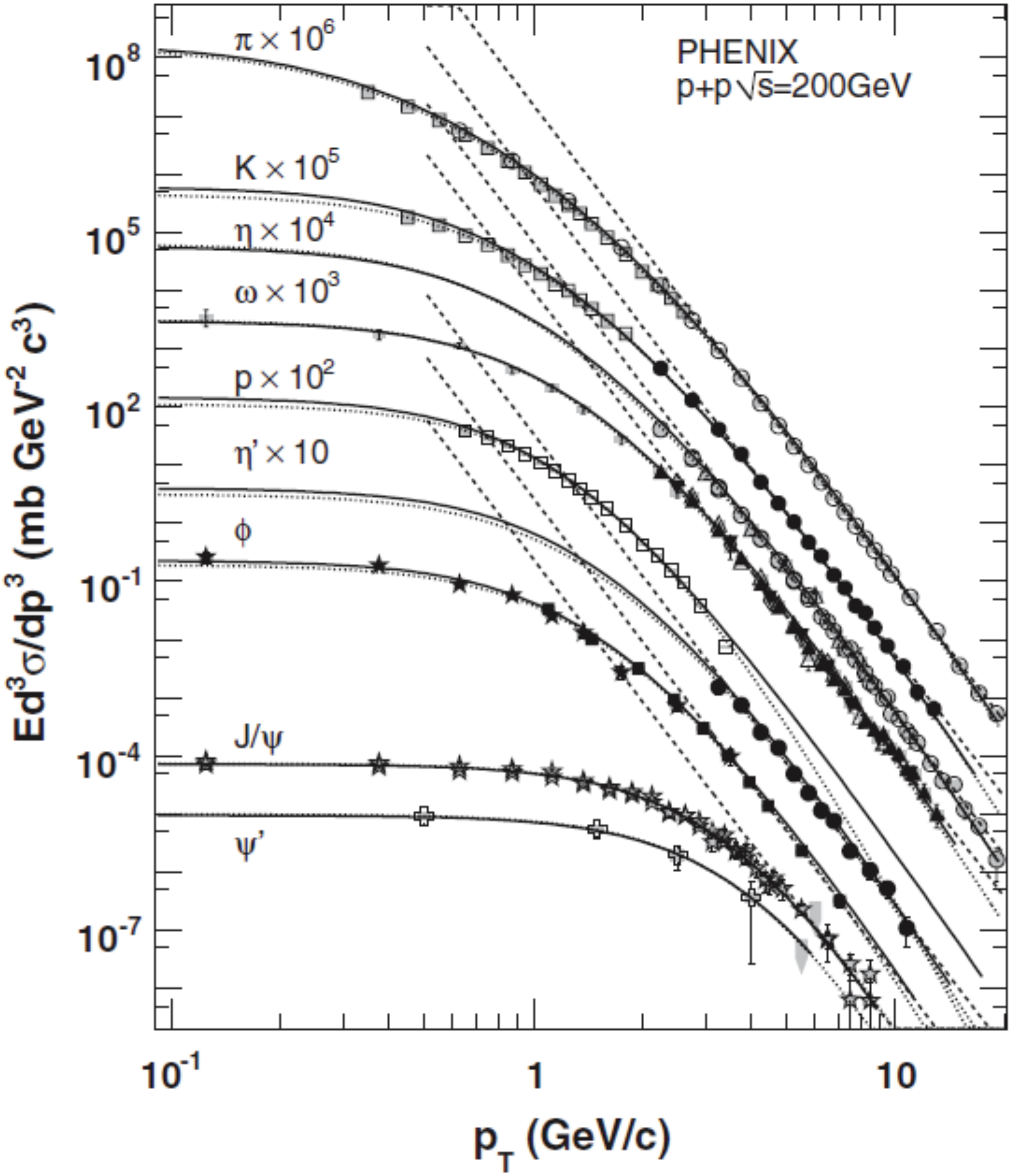
Measurement of neutral mesons in $p + p$ collisions at $\sqrt{s} = 200$ GeV and scaling properties of hadron production

A. Adare,[11] S. Afanasiev,[25] C. Aidala,[12,36] N. N. Ajitanand,[53] Y. Akiba,[47,48] H. Al-Bataineh,[42] J. Alexander,[53] K. Aoki,[30,47] L. Aphecetche,[55] R. Armendariz,[42] S. H. Aronson,[6] J. Asai,[47,48] E. T. Atomssa,[31] R. Averbeck,[54] T. C. Awes,[43] B. Azmoun,[6] V. Babintsev,[21] M. Bai,[5] G. Baksay,[17] L. Baksay,[17] A. Baldisseri,[14] K. N. Barish,[7] P. D. Barnes,[33] B. Bassalleck,[41] A. T. Basye,[1] S. Bathe,[7] S. Batsouli,[43] V. Baublis,[46] C. Baumann,[37] A. Bazilevsky,[6] S. Belikov,[6,*] R. Bennett,[54] A. Berdnikov,[50] Y. Berdnikov,[50] A. A. Bickley,[11] J. G. Boissevain,[33] H. Borel,[14] K. Boyle,[54] M. L. Brooks,[33] H. Buesching,[6] V. Bumazhnov,[21] G. Bunce,[6,48] S. Butsyk,[33,54] C. M. Camacho,[33] S. Campbell,[54] B. S. Chang,[62] W. C. Chang,[2] J.-L. Charvet,[14] S. Chernichenko,[21] J. Chiba,[26] C. Y. Chi,[12] M. Chiu,[22] I. J. Choi,[62] R. K. Choudhury,[4] T. Chujo,[58,59] P. Chung,[53] A. Churyn,[21] V. Cianciolo,[43] Z. Citron,[54] C. R. Cleven,[19] B. A. Cole,[12] M. P. Comets,[44] P. Constantin,[33] M. Csanád,[16] T. Csörgő,[27] T. Dahms,[54] S. Dairaku,[30,47] K. Das,[18] G. David,[6] M. B. Deaton,[1] K. Dehmelt,[17] H. Delagrange,[55] A. Denisov,[21] D. d'Enterria,[12,31] A. Deshpande,[48,54] E. J. Desmond,[6] O. Dietzsch,[51] A. Dion,[54] M. Donadelli,[51] O. Drapier,[31] A. Drees,[54] K. A. Drees,[5] A. K. Dubey,[61] A. Durum,[21] D. Dutta,[4] V. Dzhordzhadze,[7] Y. V. Efremenko,[43] J. Egdemir,[54] F. Ellinghaus,[11] W. S. Emam,[7] T. Engelmore,[12] A. Enokizono,[32] H. En'yo,[47,48] S. Esumi,[58] K. O. Eyser,[7] B. Fadem,[38] D. E. Fields,[41,48] M. Finger, Jr.,[8,25] M. Finger,[8,25] F. Fleuret,[31] S. L. Fokin,[29] Z. Fraenkel,[61,*] J. E. Frantz,[54] A. Franz,[6] A. D. Frawley,[18] K. Fujiwara,[47] Y. Fukao,[30,47] T. Fusayasu,[40] S. Gadrat,[34] I. Garishvili,[56] A. Glenn,[11] H. Gong,[54] M. Gonin,[31] J. Gosset,[14] Y. Goto,[47,48] R. Granier de Cassagnac,[31] N. Grau,[12,24] S. V. Greene,[59] M. Grosse Perdekamp,[22,48] T. Gunji,[10] H.-Å. Gustafsson,[35,*] T. Hachiya,[20] A. Hadj Henni,[55] C. Haegemann,[41] J. S. Haggerty,[6] H. Hamagaki,[10] R. Han,[45] H. Harada,[20] E. P. Hartouni,[32] K. Haruna,[20] E. Haslum,[35] R. Hayano,[10] M. Heffner,[32] T. K. Hemmick,[54] T. Hester,[7] X. He,[19] H. Hiejima,[22] J. C. Hill,[24] R. Hobbs,[41] M. Hohlmann,[17] W. Holzmann,[53] K. Homma,[20] B. Hong,[28] T. Horaguchi,[10,47,57] D. Hornback,[56] S. Huang,[59] T. Ichihara,[47,48] R. Ichimiya,[47] H. Iinuma,[30,47] Y. Ikeda,[58] K. Imai,[30,47] J. Imrek,[15] M. Inaba,[58] Y. Inoue,[49,47] D. Isenhower,[1] L. Isenhower,[1] M. Ishihara,[47] T. Isobe,[10] M. Issah,[53] A. Isupov,[25] D. Ivanischev,[46] B. V. Jacak,[54,†] J. Jia,[12] J. Jin,[12] O. Jinnouchi,[48] B. M. Johnson,[6] K. S. Joo,[39] D. Jouan,[44] F. Kajihara,[10] S. Kametani,[10,47,60] N. Kamihara,[47,48] J. Kamin,[54] M. Kaneta,[48] J. H. Kang,[62] H. Kanou,[47,57] J. Kapustinsky,[33] D. Kawall,[36,48] A. V. Kazantsev,[29] T. Kempel,[24] A. Khanzadeev,[46] K. M. Kijima,[20] J. Kikuchi,[60] B. I. Kim,[28] D. H. Kim,[39] D. J. Kim,[62] E. Kim,[52] S. H. Kim,[62] E. Kinney,[11] K. Kiriluk,[11] Á. Kiss,[16] E. Kistenev,[6] A. Kiyomichi,[47] J. Klay,[32] C. Klein-Boesing,[37] L. Kochenda,[46] V. Kochetkov,[21] B. Komkov,[46] M. Konno,[58] J. Koster,[22] D. Kotchetkov,[7] A. Kozlov,[61] A. Král,[13] A. Kravitz,[12] J. Kubart,[8,23] G. J. Kunde,[33] N. Kurihara,[10]

PHENIX @ RHIC


PHENIX
p+p √s=200GeV
(π⁺+π⁻)/2
π⁰→ γγ
(K⁺+K⁻)/2
K⁰S→ π⁰π⁰
η → γγ
π⁰π⁺π⁻
ω → e⁺e⁻
π⁰π⁺π⁻
π⁰γ
(p+p̄)/2
η'→ ηπ⁺π⁻
φ → e⁺e⁻
K⁺K⁻
J/ψ → e⁺e⁻
ψ'→ e⁺e⁻
π × 10⁶
K × 10⁵
ω × 10³
η × 10⁴
ψ'
J/ψ
φ
η' × 10
p × 10²
Ed³σ/dp³ (mb GeV⁻² c³)
p_T (GeV/c)

$q \simeq 1.10$



$q \simeq 1.10$

FIG. 13. The p_T spectra of various hadrons measured by PHENIX fitted to the power law fit (dashed lines) and Tsallis fit (solid lines). See text for more details.

PHYSICAL REVIEW D **87,** 114007 (2013)

Tsallis fits to p_T spectra and multiple hard scattering in pp collisions at the LHC

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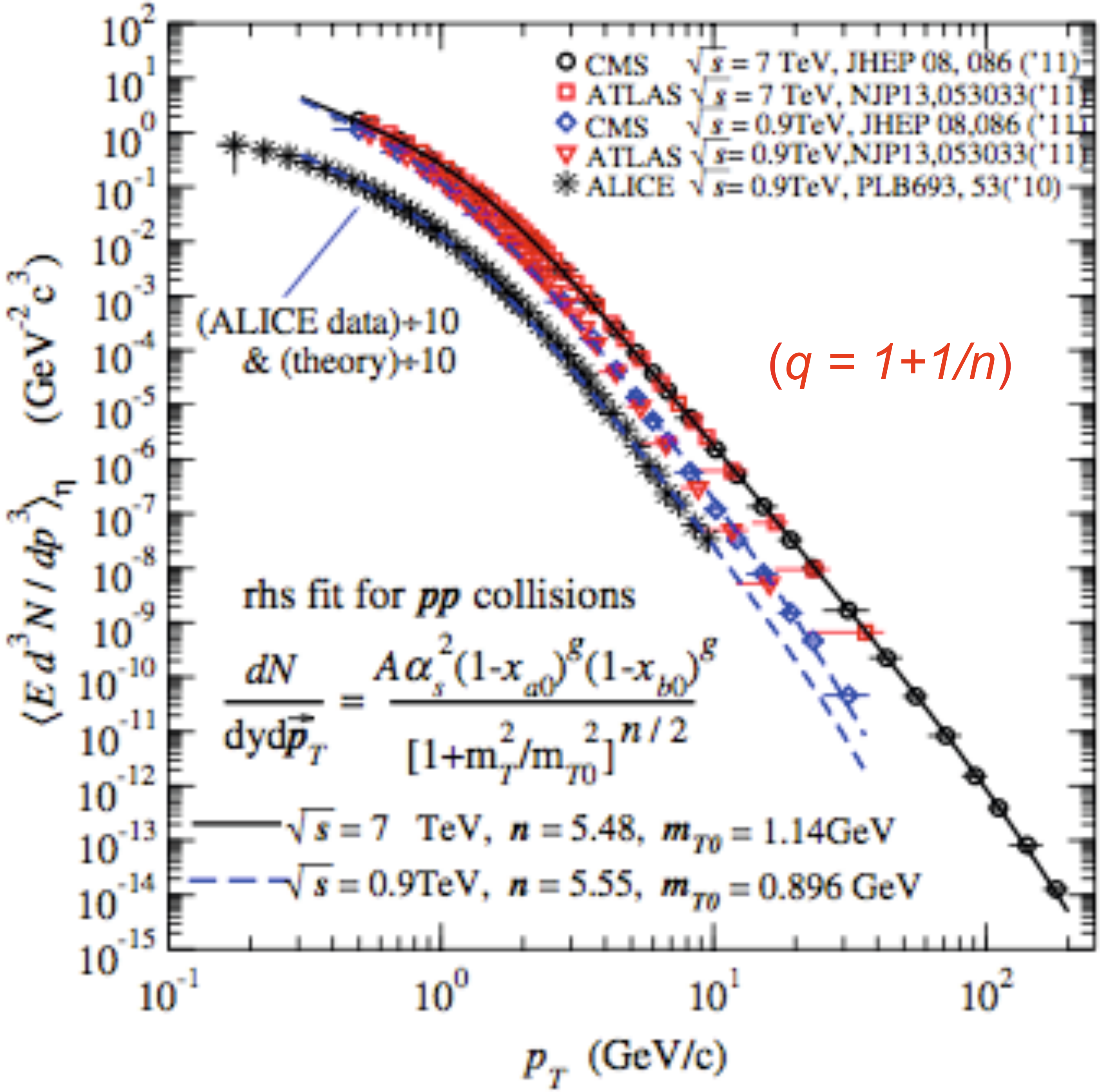
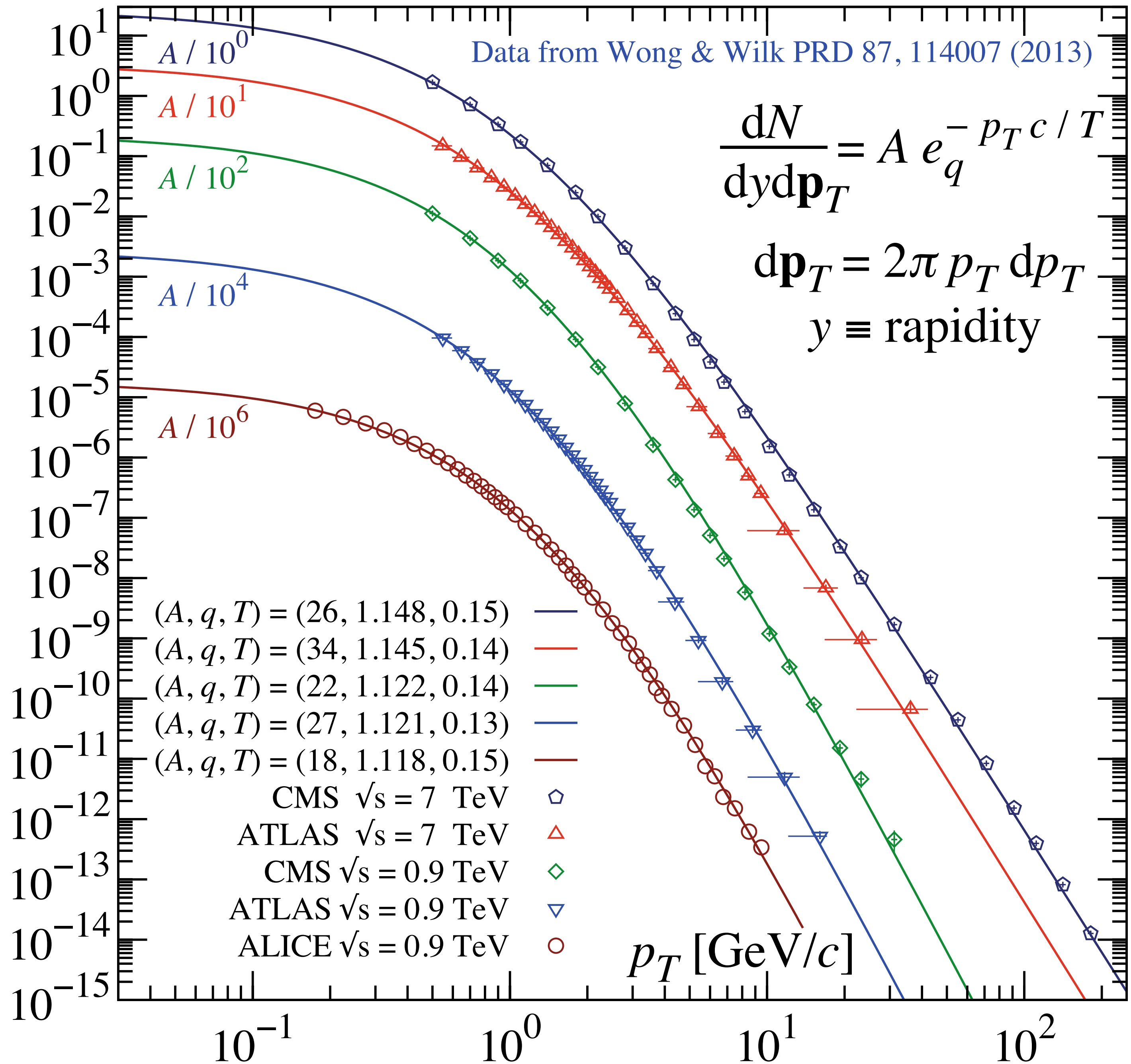
Grzegorz Wilk
National Centre for Nuclear Research, Warsaw 00-681, Poland
(Received 12 May 2013; published 5 June 2013)

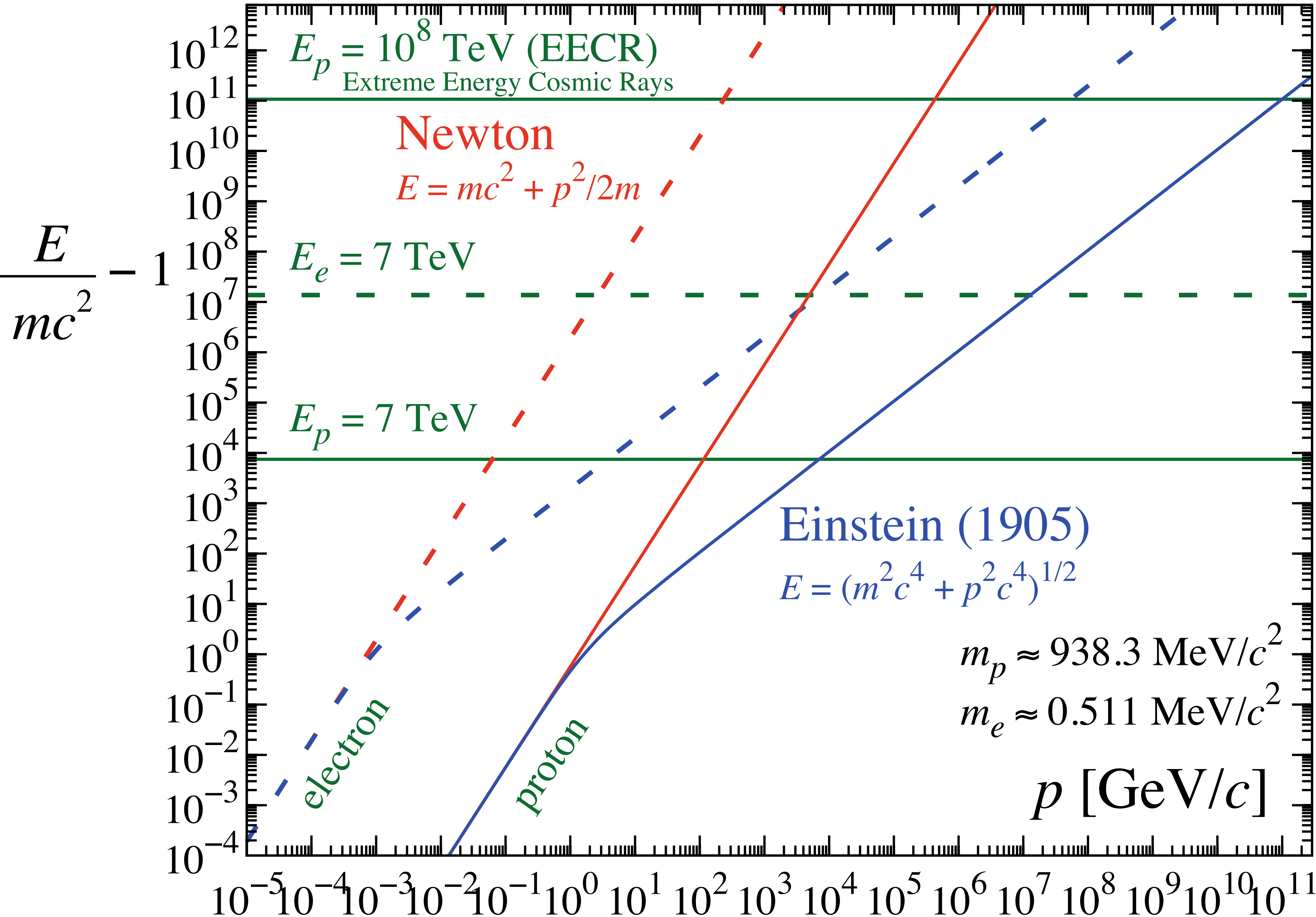
Phenomenological Tsallis fits to the CMS, ATLAS, and ALICE transverse momentum spectra of hadrons for pp collisions at LHC were recently found to extend over a large range of the transverse momentum. We investigate whether the few degrees of freedom in the Tsallis parametrization may arise from the relativistic parton-parton hard-scattering and related processes. The effects of the multiple hard-scattering and parton showering processes on the power law are discussed. We find empirically that whereas the transverse spectra of both hadrons and jets exhibit power-law behavior of $1/p_T^n$ at high p_T, the power indices n for hadrons are systematically greater than those for jets, for which $n \sim 4$–5.



CMS $\sqrt{s}$ = 7 TeV, JHEP 08, 086 ('11)
ATLAS $\sqrt{s}$ = 7 TeV, NJP13,053033('11)
CMS $\sqrt{s}$ = 0.9TeV, JHEP 08,086 ('11)
ATLAS $\sqrt{s}$= 0.9TeV,NJP13,053033('11)
ALICE $\sqrt{s}$= 0.9TeV, PLB693, 53('10)
(ALICE data)+10
& (theory)+10
(q = 1+1/n)
rhs fit for pp collisions
$\frac{dN}{dy d\vec{p}_T} = \frac{A\alpha_s^2(1-x_{a0})^g(1-x_{b0})^g}{[1+m_T^2/m_{T0}^2]^{n/2}}$
$\sqrt{s}$ = 7 TeV, n = 5.48, m_{T0} = 1.14GeV
$\sqrt{s}$ = 0.9TeV, n = 5.55, m_{T0} = 0.896 GeV
$\langle E\, d^3N / dp^3 \rangle_\eta$ (GeV^{-2}c^3)
p_T (GeV/c)



L.J.L. Cirto and C. T. (2013)



$\frac{E}{mc^2} - 1$
$E_p = 10^8$ TeV (EECR)
Extreme Energy Cosmic Rays
Newton
$E = mc^2 + p^2/2m$
$E_e = 7$ TeV
$E_p = 7$ TeV
Einstein (1905)
$E = (m^2c^4 + p^2c^4)^{1/2}$
$m_p \approx 938.3$ MeV/c^2
$m_e \approx 0.511$ MeV/c^2
electron
proton
p [GeV/c]

PHYSICAL REVIEW D VOLUME 13, NUMBER 2 15 JANUARY 1976

Black holes and thermodynamics*

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and Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, England
(Received 30 June 1975)

A black hole of given mass, angular momentum, and charge can have a large number of different unobservable internal configurations which reflect the possible different initial configurations of the matter which collapsed to produce the hole. The logarithm of this number can be regarded as the entropy of the black hole and is a measure of the amount of information about the initial state which was lost in the formation of the black hole. If one makes the hypothesis that the entropy is finite, one can deduce that the black holes must emit thermal radiation at some nonzero temperature. Conversely, the recently derived quantum-mechanical result that black holes do emit thermal radiation at temperature $\kappa\hbar/2\pi kc$, where κ is the surface gravity, enables one to prove that the entropy is finite and is equal to $c^3A/4G\hbar$, where A is the surface area of the event horizon or boundary of the black hole. Because black holes have negative specific heat, they cannot be in stable thermal equilibrium except when the additional energy available is less than 1/4 the mass of the black hole. This means that the standard statistical-mechanical canonical ensemble cannot be applied when gravitational interactions are important. Black holes behave in a completely random and time-symmetric way and are indistinguishable, for an external observer, from white holes. The irreversibility that appears in the classical limit is merely a statistical effect.

When entropy does not seem extensive

Earlier speculations about the entropy of black holes has prompted an ingenious calculation suggesting that entropy may (in special circumstances) be the same inside and outside an arbitrary boundary.

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. That, of course, is why the entropy of some substance will be quoted as so much per gram, or mole. If you then take two grams, or two moles, of the same material under the same conditions, the entropy will be twice as much. And there should be no confusion about the units; the simple Carnot definition of a change of entropy in a reversible process is the heat transfer divided by the absolute temperature, so that the units of entropy are simply those of energy divided by temperature, joules per degree (kelvin) in the SI system. The definitions of the Gibbs and Helmholtz free energies would be dimensionally discordant for that reason were it not that entropy (S) always turns up multiplied by temperature T. So much will readily be agreed.

Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? By the number of ways in which the constituents of some material (the atoms and molecules) can be rearranged without changing its properties and without energetic consequences. But now there comes a snag.

Like any extensive property, the combined entropy of two separate chunks of material should be the sum of the two entropies, but the number of rearrangements of the combined system must be the *product* of the numbers of ways in which the two parts separately can be rearranged. How to reconcile that with extensivity? By supposing entropy is proportional not to the number of rearrangements (technically called 'complexions'), but with the logarithm thereof. And because entropy decreases as disorder increases, the constant of proportionality must be a negative (real) number.

From that it follows that $S = S_0 - K \log N$, where K is a positive constant with the dimensions of entropy, N is a number (without dimensions) measuring disorder and S_0 is an arbitrary constant entropy. All that is simply a precis of the standard introductory chapter in statistical mechanics textbooks, most of which go on to show how to calculate the properties of assemblages of, say, diatomic molecules from a knowledge of their individual behaviour. Because the number of complexions of a particular state of an assemblage is invariably a function of the number (n) of molecules it contains, usually in the form of $n!$, because n is usually large and because $\log(n!)$ can then be approximated by $n \log n$, the extensive property of entropy then follows simply from the appearance of the leading factor n: entropy is proportional to the number of molecules.

That is what the textbooks say. It also makes sense of what is known of the thermodynamics of the real world. In a sample of a diatomic gas, for example, there are vibrations (one) and rotations (two) as well as three rectilinear degrees of freedom. But the problem is to tell how the energy available is distributed among the different degrees of freedom. The arithmetic simplifies marvellously because (in this case) each molecule and each of its degrees of freedom is independent. The best measure of disorder works out at $N = Z^n$, where n is the number of molecules, and where Z, which must be a

A bit of quantum mechanics goes into the argument as well, notably the notion of the density matrix — an artificially constructed operator (on quantum states) that is well suited to the discussion of systems in which one part (say the black hole) is singled out for attention while the remainder (the Universe outside it) is dealt with in less detail, perhaps because some averaging process is appropriate, or because the whole problem may not be calculable at all. (In Dirac's notation, the density matrix corresponding to some state of the whole Universe would be represented as $|1\rangle\langle 1|$, where "1" is simply the name for a particular state of the Universe.) What matters, where entropy is concerned, is that the density matrix, like all matrices, has eigenvalues from which the entropy can be calculated.

So imagine that the Universe is partitioned into two parts by means of a closed boundary of some kind and filled with a

dealt with explicitly, as other entropy calculations are made. And that could be exceedingly important.

John Maddox

Tackled by

- Jacob D. Bekenstein
- Stephen W. Hawking
- Gary W. Gibbons
- Gerard 't Hooft
- Leonard Susskind
- Michael J. Duff
- Juan M. Maldacena
- Thanu Padmanabhan
- Robert M. Wald

and many others

When entropy does not seem extensive

John Maddox, Nature **365**, 103 (1993)

Everybody who knows about entropy knows that it is an extensive property, like mass or enthalpy. [...] Of course, there is more than that to entropy, which is also a measure of disorder. Everybody also agrees on that. But how is disorder measured? [...] So why is the entropy of a black hole proportional to the square of its radius, and not to the cube of it? To its surface area rather than to its volume?

RAPID COMM

PHYSICAL REVIEW D **73,** 121701(R) (2006)

How robust is the entanglement entropy-area relation?

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We revisit the problem of finding the entanglement entropy of a scalar field on a lattice by tracing over its degrees of freedom inside a sphere. It is known that this entropy satisfies the area law—entropy proportional to the area of the sphere—when the field is assumed to be in its ground state. We show that the area law continues to hold when the scalar field degrees of freedom are in generic coherent states and a class of squeezed states. However, when excited states are considered, the entropy scales as a lower power of the area. This suggests that, for large horizons, the ground state entropy dominates, whereas entropy due to excited states gives power-law corrections. We discuss possible implications of this result to black hole entropy.

The area (as opposed to volume) proportionality of BH entropy has been an intriguing issue for decades.

PHYSICAL REVIEW D **83,** 064034 (2011)

Ideal gas in a strong gravitational field: Area dependence of entropy

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(Received 24 January 2011; published 24 March 2011)

We study the thermodynamic parameters like entropy, energy etc. of a box of gas made up of indistinguishable particles when the box is kept in various static background spacetimes having a horizon. We compute the thermodynamic variables using both statistical mechanics as well as by solving the hydrodynamical equations for the system. When the box is far away from the horizon, the entropy of the gas depends on the volume of the box except for small corrections due to background geometry. As the box is moved closer to the horizon with one (leading) edge of the box at about Planck length (L_p) away from the horizon, the entropy shows an area dependence rather than a volume dependence. More precisely, it depends on a small volume $A_\perp L_p/2$ of the box, up to an order $\mathcal{O}(L_p/K)^2$ where $A_\perp$ is the transverse area of the box and K is the (proper) longitudinal size of the box related to the distance between leading and trailing edge in the vertical direction (i.e. in the direction of the gravitational field). Thus the contribution to the entropy comes from only a fraction $\mathcal{O}(L_p/K)$ of the matter degrees of freedom and the rest are suppressed when the box approaches the horizon. Near the horizon all the thermodynamical quantities behave as though the box of gas has a volume $A_\perp L_p/2$ and is kept in a Minkowski spacetime. These effects are: (i) purely kinematic in their origin and are independent of the spacetime curvature (in the sense that the Rindler approximation of the metric near the horizon can reproduce the results) and (ii) observer dependent. When the equilibrium temperature of the gas is taken to be equal to the horizon temperature, we get the familiar $A_\perp/L_p^2$ dependence in the expression for entropy. All these results hold in a $D+1$ dimensional spherically symmetric spacetime. The analysis based on methods of statistical mechanics and the one
lead to the same result

Thus the extensive property of entropy no longer holds and one can check that it does not hold even in the weak field limit discussed above when $L \gg \lambda$ that is, when gravitational effects subdue the thermal effects along the direction of the gravitational field.

SINCE THE PIONEERING BEKENSTEIN-HAWKING RESULTS, PHYSICALLY MEANINGFUL EVIDENCE HAS ACCUMULATED (e.g., HOLOGRAPHIC PRINCIPLE) WHICH MANDATES THAT

$$\ln W_{black\ hole} \propto \text{AREA}$$

THIS IS PERFECTLY ADMISSIBLE AND MOST PROBABLY CORRECT.

HOWEVER,

IS THIS QUANTITY THE THERMODYNAMICAL ENTROPY???

Eur. Phys. J. C (2013) 73:2487
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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Black hole thermodynamical entropy

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[2]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

Abstract As early as 1902, Gibbs pointed out that systems whose partition function diverges, e.g. gravitation, lie outside the validity of the Boltzmann–Gibbs (BG) theory. Consistently, since the pioneering Bekenstein–Hawking results, physically meaningful evidence (e.g., the holographic principle) has accumulated that the BG entropy S_{BG} of a $(3+1)$ black hole is proportional to its area L^2 (L being a characteristic linear length), and not to its volume L^3. Similarly it exists the *area law*, so named because, for a wide class of strongly quantum-entangled d-dimensional systems, S_{BG} is proportional to $\ln L$ if $d=1$, and to L^{d-1} if $d>1$, instead of being proportional to L^d $(d \geq 1)$. These results violate the extensivity of the thermodynamical entropy of a d-dimensional system. This thermodynamical inconsistency disappears if we realize that the thermodynamical entropy of such nonstandard systems is *not* to be identified with the BG *additive* entropy but with appropriately generalized *nonadditive* entropies. Indeed, the celebrated usefulness of the BG entropy is founded on hypothesis such as relatively weak probabilistic correlations (and their connections to ergodicity, which by no means can be assumed as a general rule of nature). Here we introduce a generalized entropy which, for the Schwarzschild black hole and the area law, can solve the thermodynamic puzzle.

Various arguments (phenomenological, holographic principle,
string theory, area law, etc) yield

$$S_{BG}(L) \equiv k_B \ln W(L) \propto L^{d-1} \quad (d > 1)$$

hence

$$W(L) \propto \Phi(L)\, \nu^{L^{d-1}} \quad \left(\text{with } \lim_{L\to\infty} \frac{\ln \Phi(L)}{L^{d-1}} = 0;\ \text{e.g., } \Phi(L) \propto L^{\rho} \right)$$

hence, for $d > 1$, the entropy which is extensive is S_δ with $\delta = \dfrac{d}{d-1}$

i.e., $$S_{\delta=d/(d-1)}(L) = k_B \sum_{i=1}^{W(L)} p_i \left(\ln \frac{1}{p_i} \right)^{\frac{d}{d-1}} \propto L^d \quad (d > 1)$$

Consequently $$S_{\delta=3/2}^{black\ hole}(L) = k_B \sum_{i=1}^{W(N)} p_i \left(\ln \frac{1}{p_i} \right)^{\frac{3}{2}} \propto L^3 \quad !!!$$

C.T. and L.J.L. Cirto, Eur. Phys. J. C **73**, 2487 (2013)

SYSTEMS $W(N)$	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)
$\sim \mu^N$ $(\mu > 1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$\sim N^\rho$ $(\rho > 0)$	NONEXTENSIVE	EXTENSIVE	NONEXTENSIVE
$\sim \nu^{N^\gamma}$ $(\nu > 1;$ $0 < \gamma < 1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE

PRL **106,** 140601 (2011) PHYSICAL REVIEW LETTERS week ending 8 APRIL 2011

Nonlinear Relativistic and Quantum Equations with a Common Type of Solution

F. D. Nobre,[1,*] M. A. Rego-Monteiro,[1] and C. Tsallis[1,2]

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[2]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*

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Generalizations of the three main equations of quantum physics, namely, the Schrödinger, Klein-Gordon, and Dirac equations, are proposed. Nonlinear terms, characterized by exponents depending on an index q, are considered in such a way that the standard linear equations are recovered in the limit $q \to 1$. Interestingly, these equations present a common, solitonlike, traveling solution, which is written in terms of the q-exponential function that naturally emerges within nonextensive statistical mechanics. In all cases, the well-known Einstein energy-momentum relation is preserved for arbitrary values of q.

See also:

R.N. Costa Filho, M.P. Almeida, G.A. Farias and J.S. Andrade, PRA **84**, 050102(R) (2011)
F.D. Nobre, M.A. Rego-Monteiro and C. T., EPL **97**, 41001 (2012)
S.H. Mazharimousavi, Phys Rev A **85**, 034102 (2012)
A.R. Plastino and C. T., J Math Phys. **54**, 041505 (2013)
R.N. Costa Filho, G. Alencar, B.S. Skagerstam and J.S. Andrade, EPL **101**, 10009 (2013)
M.A. Rego-Monteiro and F.D. Nobre, Phys Rev A **88**, 032105 (2013)
B.G. Costa and E.P. Borges, preprint (2013)

q − generalized Schroedinger equation

(quantum non-relativistic spinless free particle)

$$i\hbar \frac{\partial}{\partial t}\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right] = -\frac{1}{2-q}\frac{\hbar^2}{2m}\nabla^2\left[\frac{\Phi\left(\vec{x},t\right)}{\Phi_0}\right]^{2-q} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi\left(\vec{x},t\right) = \Phi_0 \; e_q^{i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \Phi_0 \; e_q^{i\left(\vec{k}\,.\,\vec{x}-\omega t\right)}$$

with

$$E = \frac{p^2}{2m} \quad \text{(Newtonian relation!)}$$

$$E = \hbar\omega \quad \text{(Planck relation!)}$$

$$p = \hbar k \quad \text{(de Broglie relation!)}$$

$\forall q$

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

q-generalized Klein-Gordon equation:

(quantum relativistic spinless free particle: e.g., mesons π)

$$\nabla^2 \Phi(\vec{x},t) = \frac{1}{c^2}\frac{\partial^2 \Phi(\vec{x},t)}{\partial t^2} + q\,\frac{m^2c^2}{\hbar^2}\Phi(\vec{x},t)\left[\frac{\Phi(\vec{x},t)}{\Phi_0}\right]^{2(q-1)} \qquad (q \in R)$$

Its exact solution is given by

$$\Phi(\vec{x},t) = \Phi_0\; e_q^{i(\vec{p}\,.\,\vec{x}-Et)/\hbar} = \Phi_0\; e_q^{i(\vec{k}\,.\,\vec{x}-\omega\, t)}$$

with

$$E^2 = p^2c^2 + m^2c^4 \qquad (\forall q) \qquad \text{(Einstein relation!)}$$

Particular case: $m = 0 \;\Rightarrow\; q$-plane waves

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

q-generalized Dirac equation:

(quantum relativistic spin $1/2$ matter and anti-matter free particles: e.g., electron and positron)

$$i\hbar \frac{\partial\, \Phi(\vec{x},t)}{\partial t} + i\hbar c(\vec{\alpha}.\vec{\nabla})\Phi(\vec{x},t) = \beta mc^2 A^{(q)}(\vec{x},t)\,\Phi(\vec{x},t) \qquad (q \in R)$$

with

$$\vec{\alpha} \equiv \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}; \quad \beta \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (4\times 4 \text{ matrices})$$

$$A_{ij}^{(q)}(\vec{x},t) \equiv \delta_{ij}\left[\frac{\Phi_j(\vec{x},t)}{a_j}\right]^{q-1} \qquad \left(A_{ij}^{(1)}(\vec{x},t) = \delta_{ij}\right) \qquad (4\times 4 \text{ matrix})$$

where $\{a_j\}$ are complex constants.

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

Its exact solution is given by

$$\Phi\left(\vec{x},t\right) \equiv \begin{pmatrix} \Phi_1\left(\vec{x},t\right) \\ \Phi_2\left(\vec{x},t\right) \\ \Phi_3\left(\vec{x},t\right) \\ \Phi_4\left(\vec{x},t\right) \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{i\left(\vec{p}\,.\,\vec{x}-Et\right)/\hbar} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} e_q^{i\left(\vec{k}\,.\,\vec{x}-\omega\, t\right)}$$

with $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$ being the same $\forall q$

hence

$$E^2 = p^2c^2 + m^2c^4 \quad (q \in R)$$ (Einstein relation!)

F.D. Nobre, M.A. Rego-Monteiro and C. T., Phys Rev Lett **106**, 140601 (2011)

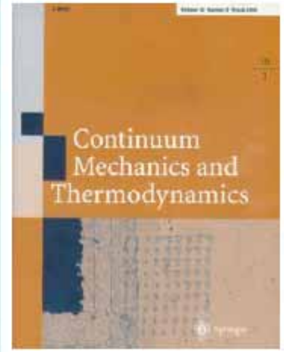
BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

1999

2001

2002

2002



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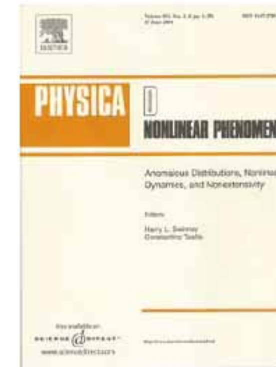

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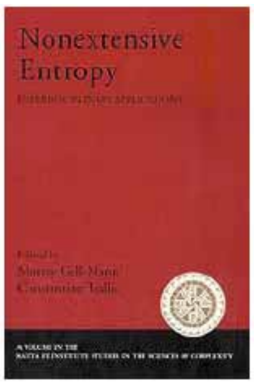

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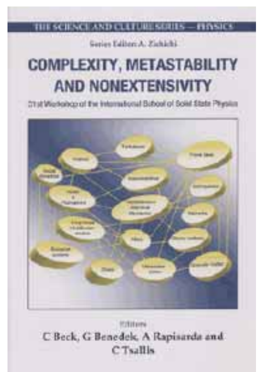

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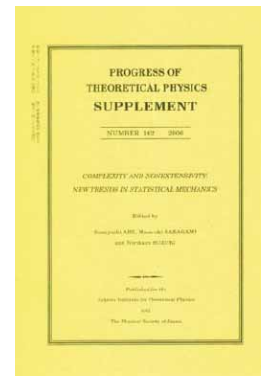

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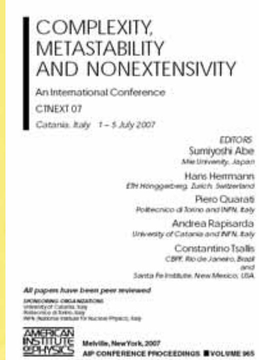

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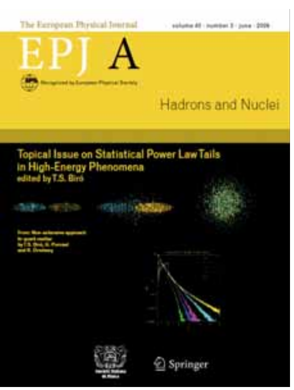

2009

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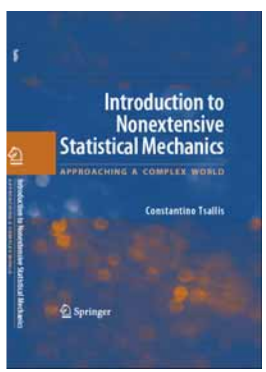

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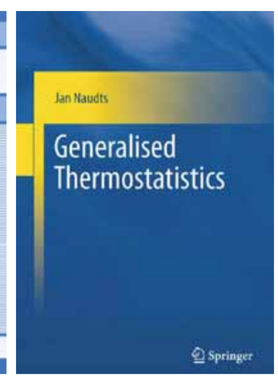

2011

2011

2012

Full bibliography (regularly updated):

http://tsallis.cat.cbpf.br/biblio.htm

4256 articles by 6077 scientists from 75 countries

[15 September 2013]

CONTRIBUTORS (4256 MANUSCRIPTS) [Updated 15 September 2013]

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JAPAN	298	ISRAEL	35	SLOVAK	8	GEORGIA	1
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RUSSIA	212	BELGIUM	22	ESTONIA	7	JORDAN	1
SPAIN	204	NETHERLANDS	20	IRELAND	7	QATAR	1
INDIA	197	BULGARIA	19	NEW ZEALAND	7	SRI LANKA	1
SWITZERLAND	189	CROATIA	16	SAUDI ARABIA	7	UZBEKISTAN	1
TURKEY	102	PAKISTAN	16	SINGAPORE	7		
ARGENTINA	101	ALGERIA	14	BANGLADESH	5		
GREECE	96	COLOMBIA	14	ARMENIA	4		
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CANADA	66	DENMARK	12	THAILAND	4		
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IRAN	56	ROMENIA	12	BELARUS	3		

75 COUNTRIES 6077 SCIENTISTS

Introduction to Nonextensive Statistical Mechanics

APPROACHING A COMPLEX WORLD

Constantino Tsallis

Springer

Physics Reports 478 (2009) 1–69

Contents lists available at ScienceDirect

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The large deviation approach to statistical mechanics

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ABSTRACT

The theory of large deviations is concerned with the exponential decay of probabilities of large fluctuations in random systems. These probabilities are important in many fields of study, including statistics, finance, and engineering, as they often yield valuable information about the large fluctuations of a random system around its most probable state or trajectory. In the context of equilibrium statistical mechanics, the theory of large deviations provides exponential-order estimates of probabilities that refine and generalize Einstein's theory of fluctuations. This review explores this and other connections between large deviation theory and statistical mechanics, in an effort to show that the mathematical language of statistical mechanics is the language of large deviation theory. The first part of the review presents the basics of large deviation theory, and works out many of its classical applications related to sums of random variables and Markov processes. The second part goes through many problems and results of statistical mechanics, and shows how these can be formulated and derived within the context of large deviation theory. The problems and results treated cover a wide range of physical systems, including equilibrium many-particle systems, noise-perturbed dynamics, nonequilibrium systems, as well as multifractals, disordered systems, and chaotic systems. This review also covers

	PHYSICS (Statistical mechanics)	**MATHEMATICS** (Large deviation theory)
$q=1$ (quasi-independent)	$p_N \propto e^{-\beta H_N} = e^{-\left[\beta \frac{H_N}{N}\right]N}$	$P_N(x) \sim e^{-N\, r(x)}$ $(N \to \infty)$
$q \neq 1$ (strongly correlated)	$p_N \propto e_q^{-\beta_q H_N} = e_q^{-\left[(\beta_q N^*) \frac{H_N}{N N^*}\right]N}$	$P_N(x) \sim e_q^{-N\, r_q(x)}$ $(N \to \infty)$

G. Ruiz and C. T., Phys Lett A **376**, 2451 (2012)

Physics Letters A 376 (2012) 2451–2454

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Physics Letters A

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Towards a large deviation theory for strongly correlated systems

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ABSTRACT

A large-deviation connection of statistical mechanics is provided by N independent binary variables, the $(N \to \infty)$ limit yielding Gaussian distributions. The probability of $n \neq N/2$ out of N throws is governed by e^{-Nr}, r related to the entropy. Large deviations for a strong correlated model characterized by indices (Q, γ) are studied, the $(N \to \infty)$ limit yielding Q-Gaussians ($Q \to 1$ recovers a Gaussian). Its large deviations are governed by $e_q^{-Nr_q}$ ($\propto 1/N^{1/(q-1)}$, $q > 1$), $q = (Q-1)/(\gamma[3-Q]) + 1$. This illustration opens the door towards a large-deviation foundation of nonextensive statistical mechanics.

Physics Letters A 377 (2013) 491–495

Contents lists available at SciVerse ScienceDirect

Physics Letters A

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Comment

Reply to Comment on "Towards a large deviation theory for strongly correlated systems"

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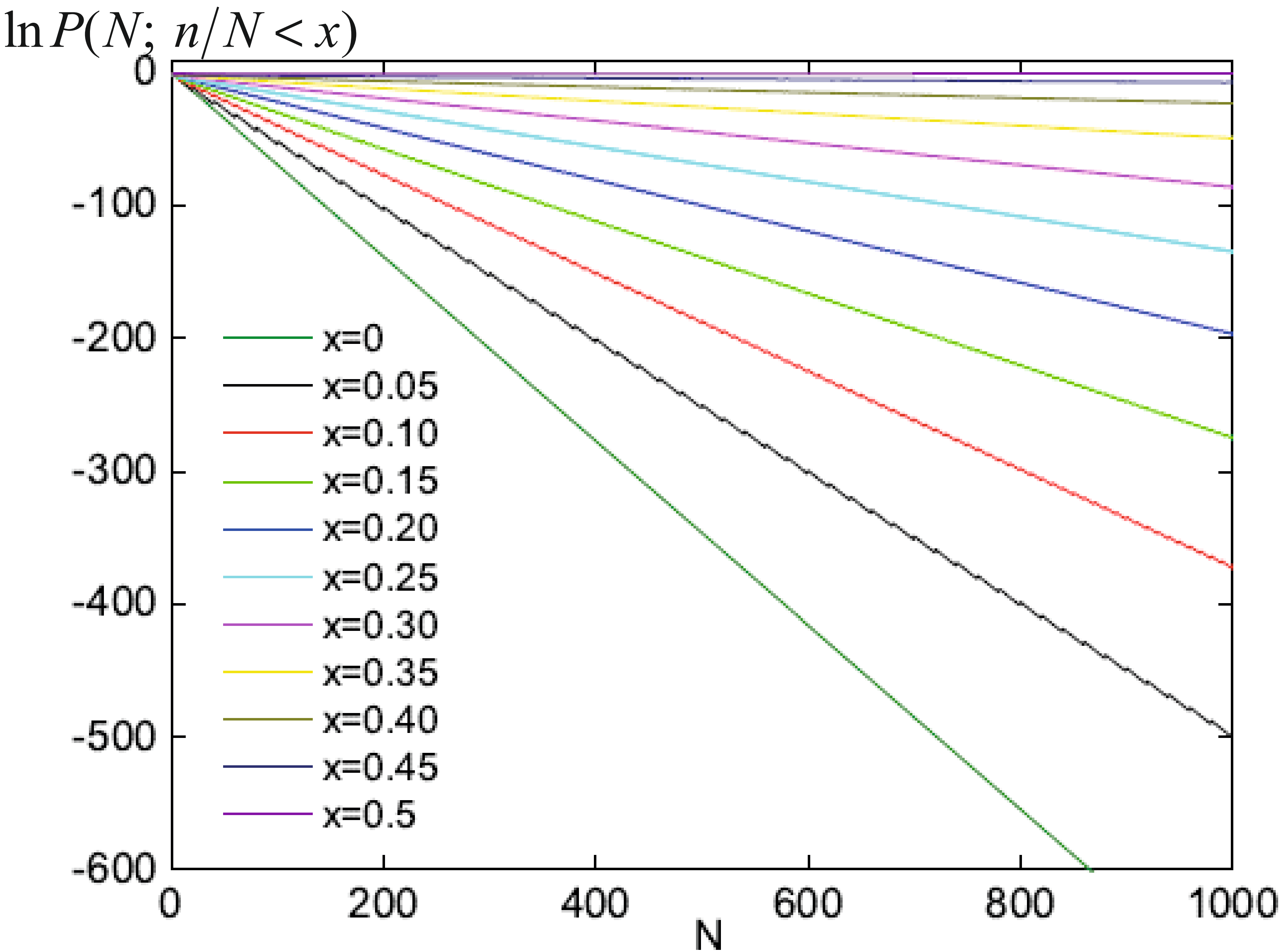
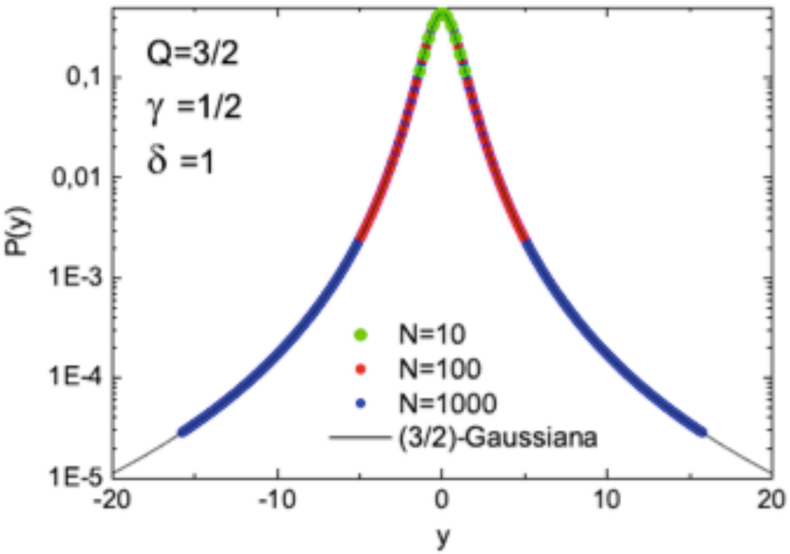
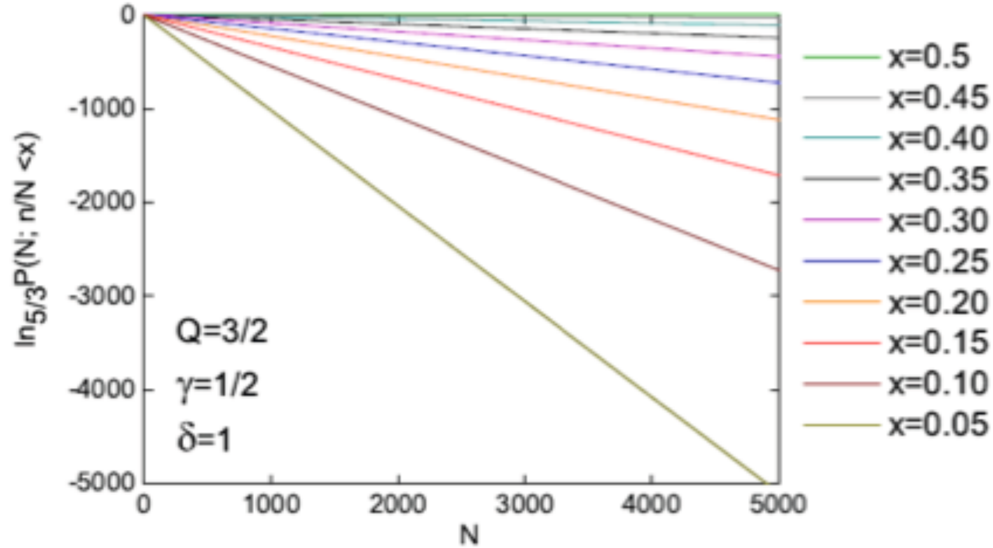
ABSTRACT

The computational study commented by Touchette opens the door to a desirable generalization of standard large deviation theory for special, though ubiquitous, correlations. We focus on three inter-related aspects: (i) numerical results strongly suggest that the standard exponential probability law is asymptotically replaced by a power-law dominant term; (ii) a subdominant term appears to reinforce the thermodynamically extensive entropic nature of q-generalized rate function; (iii) the correlations we discussed, correspond to Q-Gaussian distributions, differing from Lévy's, except in the case of Cauchy–Lorentz distributions. Touchette has agreeably discussed point (i), but, unfortunately, points (ii) and (iii) escaped to his analysis. Claiming the absence of connection with q-exponentials is unjustified.

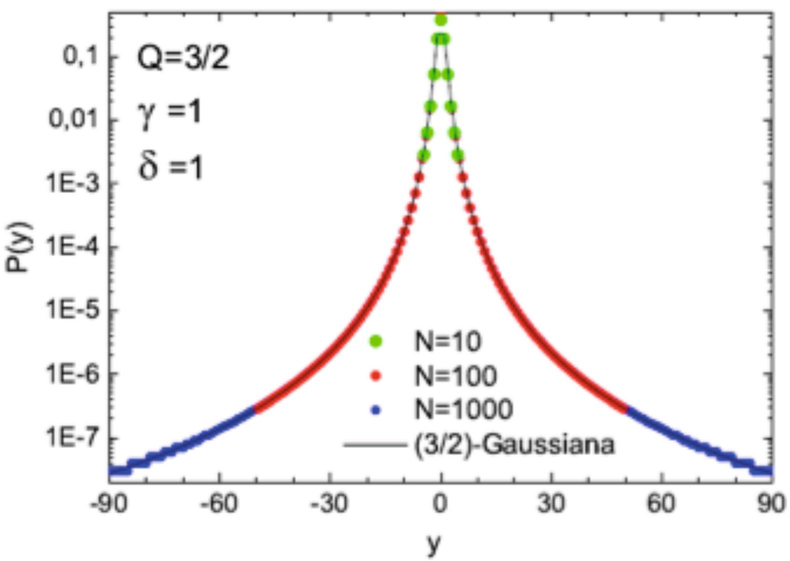


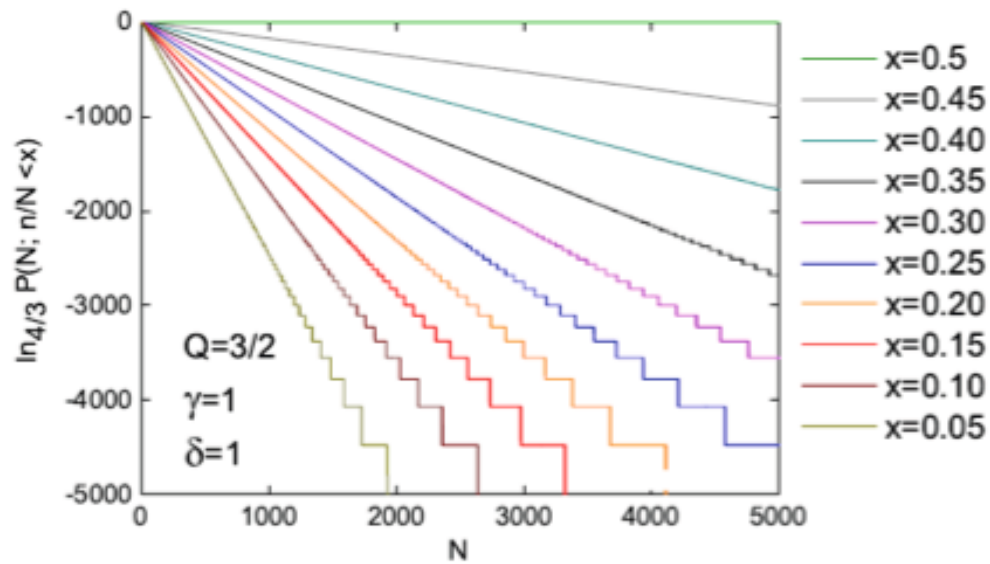
G. Ruiz and C. T., Phys Lett A **376**, 2451 (2012)



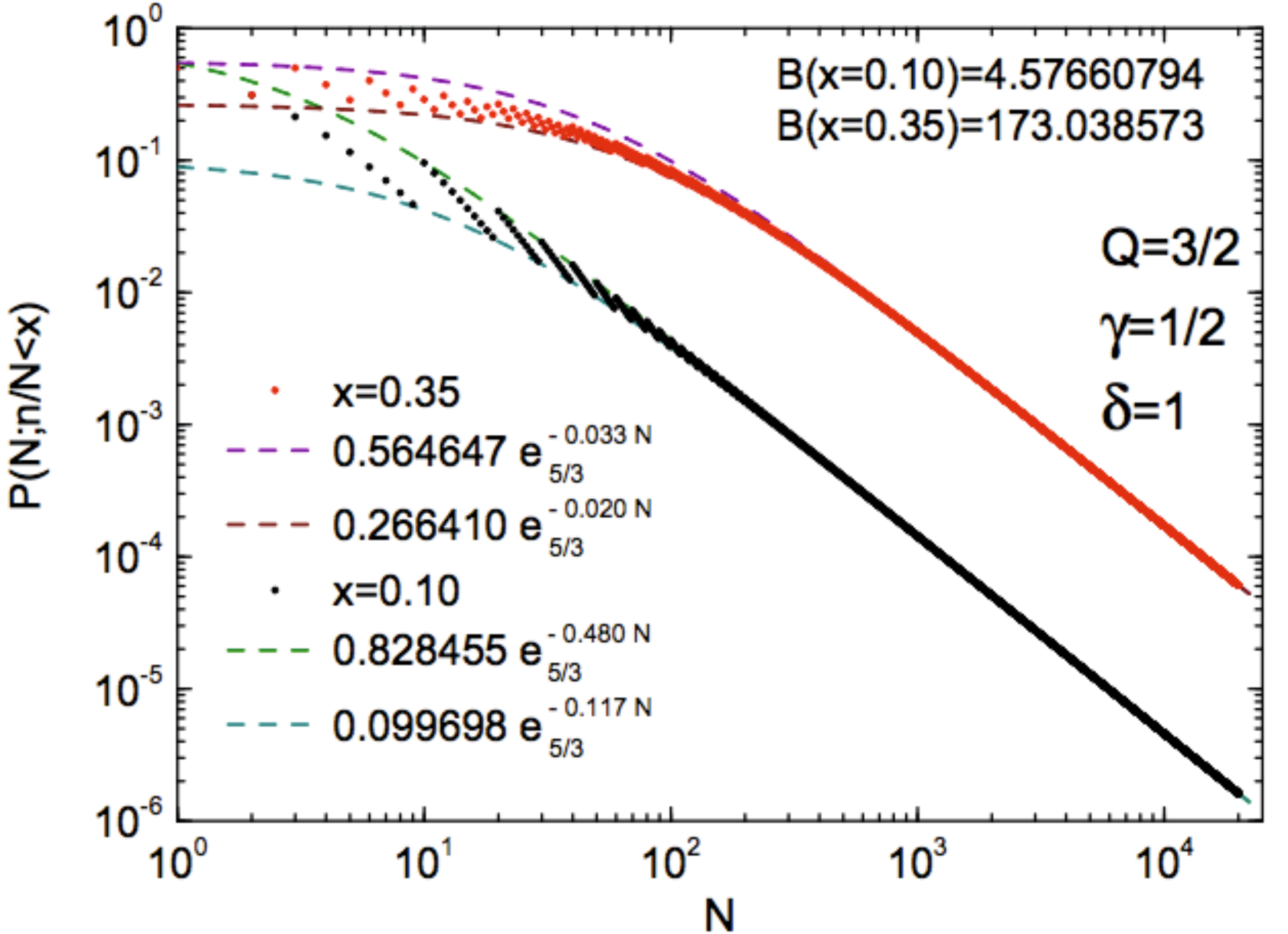


$$q = 1 + \frac{Q-1}{\gamma(3-Q)}$$

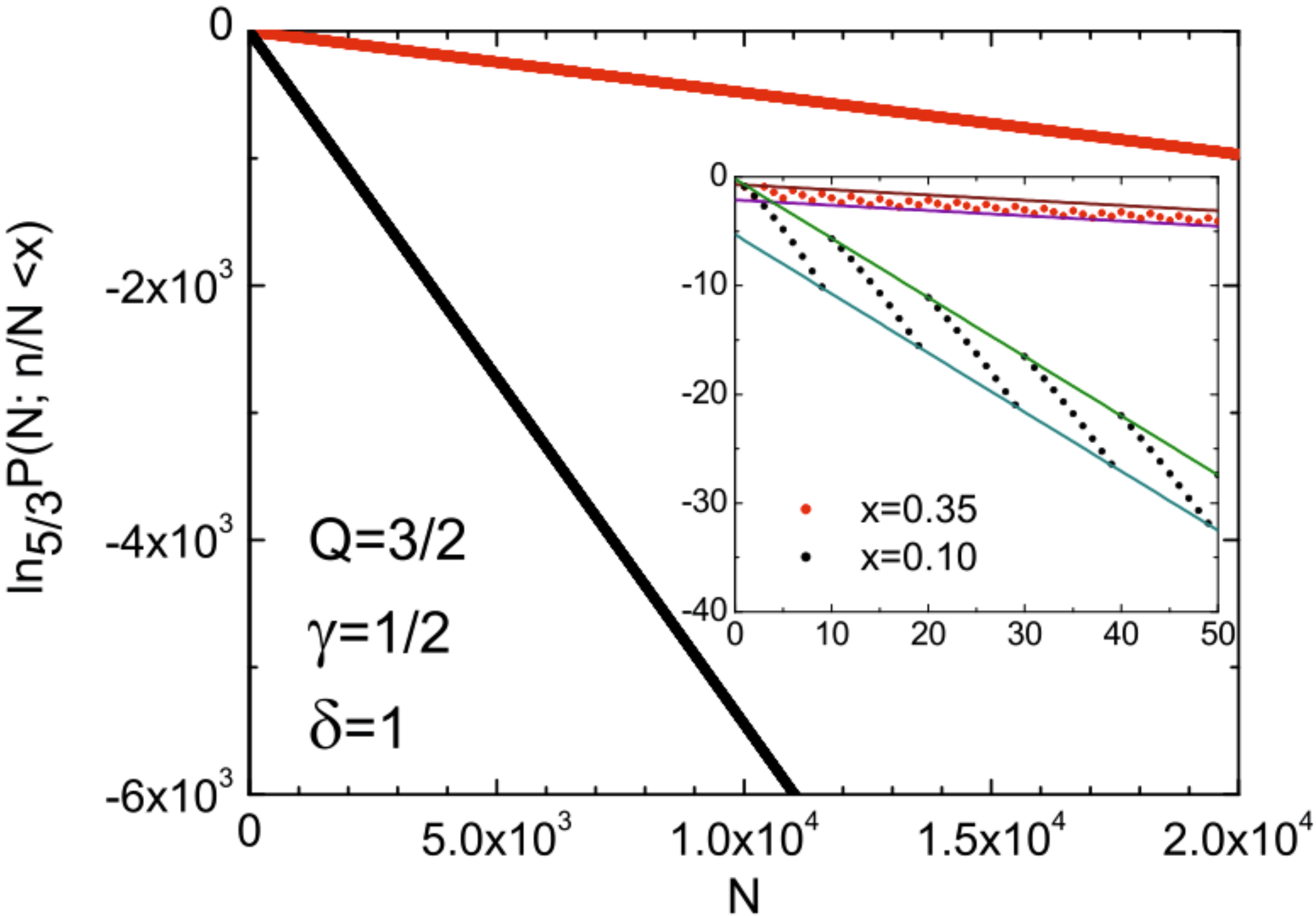




G. Ruiz and C. T., Phys Lett A **376**, 2451 (2012)



G. Ruiz and C. T., Phys Lett A **377**, 491 (2013)



G. Ruiz and C. T., Phys Lett A **377**, 491 (2013)

BOLTZMANN-GIBBS STATISTICAL MECHANICS $(q=1)$

Additive entropy $\left[S_{BG}(A+B)=S_{BG}(A)+S_{BG}(B)\right]$

Linear Fokker-Planck equation

Linear Fourier transform

$$\frac{dy}{dx}=ay \Rightarrow y=e^{ax}$$

NONEXTENSIVE STATISTICAL MECHANICS $(q \neq 1)$

Nonadditive entropy $\left[\frac{S_{BG}(A+B)}{k}=\frac{S_{BG}(A)}{k}+\frac{S_{BG}(B)}{k}+(1-q)\frac{S_{BG}(A)}{k}\frac{S_{BG}(B)}{k}\right]$

Nonlinear Fokker-Planck equation

Nonlinear q-Fourier transform

$$\frac{dy}{dx}=ay^{q} \Rightarrow y=[1+(1-q)ax]^{\frac{1}{1-q}} \equiv e_q^{ax}$$

BOLTZMANN-GIBBS STATISTICAL MECHANICS

(Maxwell 1860, Boltzmann 1872, Gibbs ≤ 1902)

Entropy $$S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i$$

Internal energy $$U_{BG} = \sum_{i=1}^{W} p_i E_i$$

Equilibrium distribution $$p_i = e^{-\beta E_i} / Z_{BG} \quad \left(Z_{BG} \equiv \sum_{j=1}^{W} e^{-\beta E_j} \right)$$

Paradigmatic differential equation $$\left.\begin{aligned} \frac{dy}{dx} &= ay \\ y(0) &= 1 \end{aligned}\right\} \Rightarrow y = e^{ax}$$

	x	a	$y(x)$
Equilibrium distribution	E_i	$-\beta$	$Z\,p(E_i)$
Sensitivity to initial conditions	t	λ	$\xi \equiv \lim_{\Delta x(0)\to 0} \frac{\Delta x(t)}{\Delta x(0)} = e^{\lambda t}$
Typical relaxation of observable O	t	$-1/\tau$	$\Omega \equiv \frac{O(t)-O(\infty)}{O(0)-O(\infty)} = e^{-t/\tau}$

S_{BG} → additive, concave, Lesche-stable, finite entropy production

NONEXTENSIVE STATISTICAL MECHANICS

(C. T. 1988, E.M.F. Curado and C. T. 1991, C. T., R.S. Mendes and A.R. Plastino 1998)

Entropy $$S_q = k\left(1 - \sum_{i=1}^{W} p_i^q\right)/(q-1)$$

Internal energy $$U_q = \sum_{i=1}^{W} p_i^q E_i \Big/ \sum_{j=1}^{W} p_j^q$$

Stationary state distribution $$p_i = e_q^{-\beta_q (E_i - U_q)} / Z_q \qquad \left(Z_q \equiv \sum_{j=1}^{W} e_q^{-\beta_q (E_j - U_q)} \right)$$

Paradigmatic differential equation $$\left.\begin{array}{l} \dfrac{dy}{dx} = a\, y^{q} \\ y(0) = 1 \end{array}\right\} \Rightarrow \quad y = e_q^{ax} \equiv [1 + (1-q)ax]^{\frac{1}{1-q}}$$

	x	a	$y(x)$
Stationary state distribution	E_i	$-\beta_{q_{stat}}$	$Z_{q_{stat}}\, p(E_i)$ (typically $q_{stat} \geq 1$)
Sensitivity to initial conditions	t	$\lambda_{q_{sen}}$	$\xi = e_{q_{sen}}^{\lambda_{q_{sen}}\, t}$ (typically $q_{sen} \leq 1$)
Typical relaxation of observable O	t	$-1/\tau_{q_{rel}}$	$\Omega = e_{q_{rel}}^{-t/\tau_{q_{rel}}}$ (typically $q_{rel} \geq 1$)

$S_q \rightarrow$ nonadditive, concave, Lesche-stable, finite entropy production

C. T., Physica A **340**,1 (2004)

Prediction of the q - triplet: C. T., Physica A **340**,1 (2004)

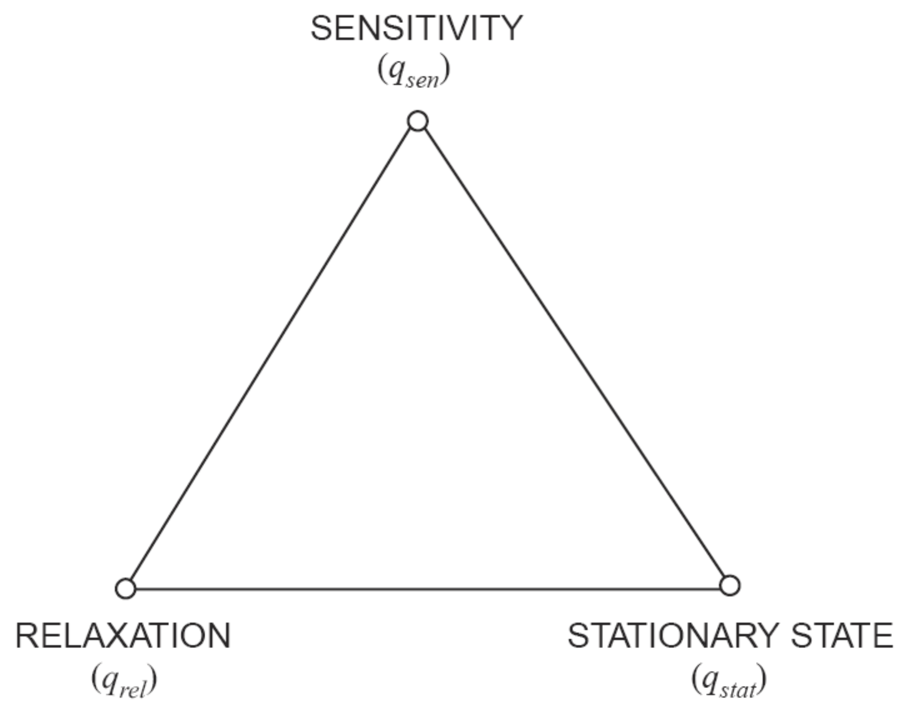


Fig. 2. The triangle of the basic values of q, namely those associated with sensitivity to the initial conditions, relaxation and stationary state. For the most relevant situations we expect $q_{sen} \leqslant 1$, $q_{rel} \geqslant 1$ and $q_{stat} \geqslant 1$. These indices are presumably inter-related since they all descend from the particular dynamical exploration that the system does of its full phase space. For example, for long-range Hamiltonian systems characterized by the decay exponent α and the dimension d, it could be that q_{stat} decreases from a value above unity (e.g., 2 or $\frac{3}{2}$) to unity when α/d increases from zero to unity. For such systems one expects relations like the (particularly simple) $q_{stat} = q_{rel} = 2 - q_{sen}$ or similar ones. In any case, it is clear that, for $\alpha/d > 1$ (i.e., when BG statistics is known to be the correct one), one has $q_{stat} = q_{rel} = q_{sen} = 1$. All the weakly chaotic systems focused on here are expected to have well defined values for q_{sen} and q_{rel}, but only those associated with a Hamiltonian are expected to *also* have a well defined value for q_{stat}.

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Physica A 356 (2005) 375–384

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Triangle for the entropic index q of non-extensive statistical mechanics observed by Voyager 1 in the distant heliosphere

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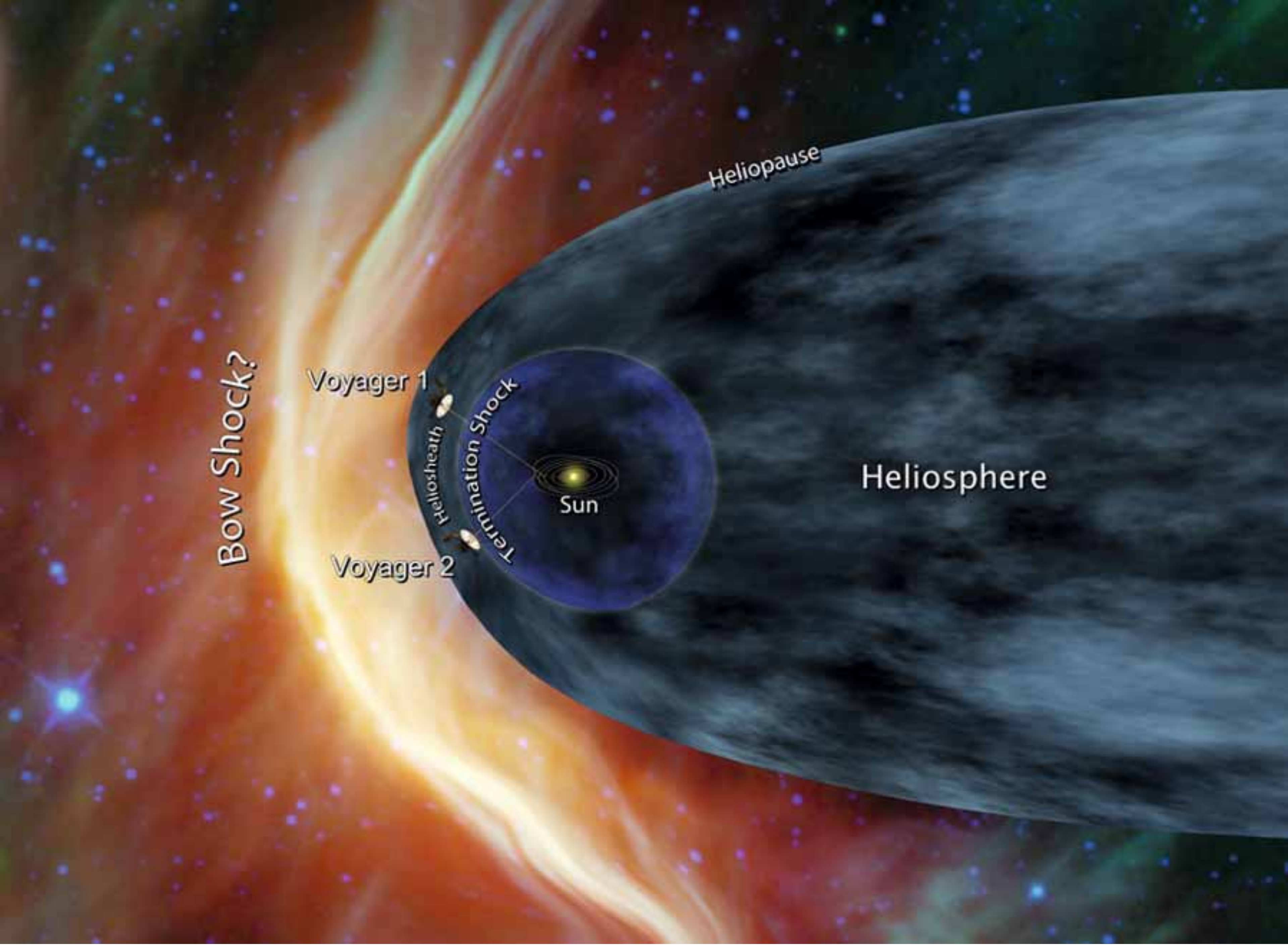
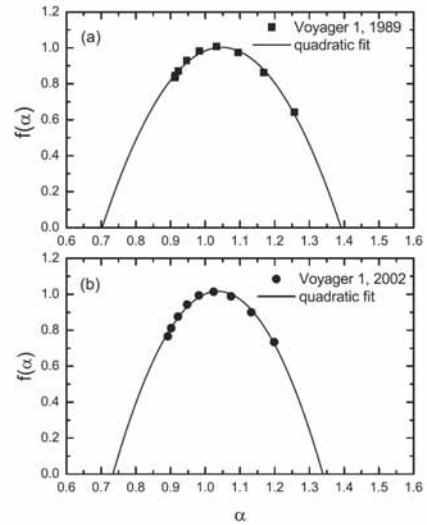
Received 10 June 2005
Available online 11 July 2005

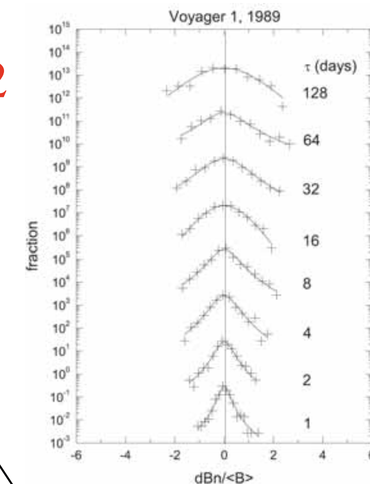

Heliopause
Bow Shock?
Voyager 1
Termination Shock
Heliosheath
Sun
Heliosphere
Voyager 2

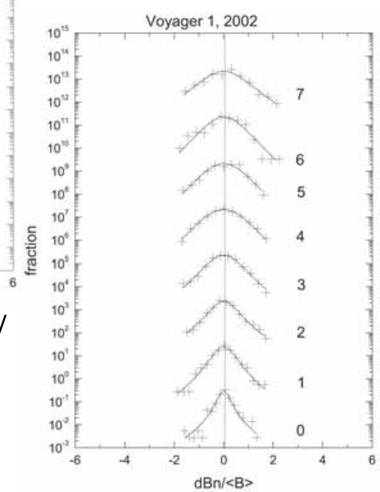
SOLAR WIND: Magnetic Field Strength

L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center; Physica A **356**, 375 (2005)

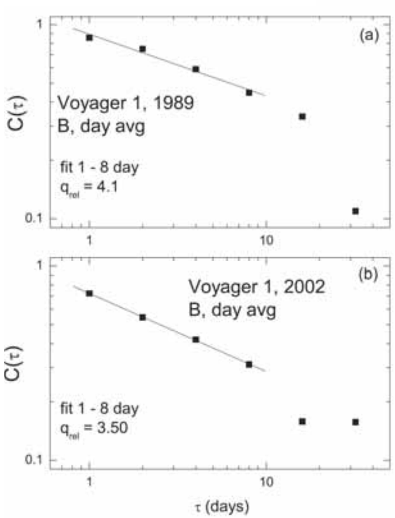
[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; daily averages]







$$q_{sen} = -0.6 \pm 0.2$$



$$q_{rel} = 3.8 \pm 0.3 \qquad q_{stat} = 1.75 \pm 0.06$$

$$q_{sens} < 1 < q_{stat} < q_{rel}$$

Asymptotically scale-invariant occupancy of phase space makes the entropy S_q extensive

Constantino Tsallis*†‡, Murray Gell-Mann*‡, and Yuzuru Sato*

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Contributed by Murray Gell-Mann, July 25, 2005

Phase space can be constructed for N equal and distinguishable subsystems that could be probabilistically either *weakly* correlated or *strongly* correlated. If they are locally correlated, we expect the Boltzmann–Gibbs entropy $S_{BG} = -k \sum_i p_i \ln p_i$ to be *extensive*, i.e., $S_{BG}(N) \propto N$ for $N \to \infty$. In particular, if they are independent, S_{BG} is *strictly additive*, i.e., $S_{BG}(N) = NS_{BG}(1), \forall N$. However, if the subsystems are globally correlated, we expect, for a vast class of systems, the entropy $S_q = k[1 - \sum_i p_i^q]/(q - 1)$ (with $S_1 = S_{BG}$) for some special value of $q \neq 1$ to be the one which is extensive [i.e., $S_q(N) \propto N$ for $N \to \infty$]. Another concept which is relevant is strict or asymptotic *scale-freedom* (or *scale-invariance*), defined as the situation for which all marginal probabilities of the N-system coincide or asymptotically approach (for $N \to \infty$) the joint probabilities of the $(N - 1)$-system. If each subsystem is a binary one, scale-freedom is guaranteed by what we hereafter refer to as the *Leibnitz rule*, i.e., the sum of two successive joint probabilities of the N-system coincides or asymptotically approaches the corresponding joint probability of the $(N - 1)$-system. The kinds of interplay of these various concepts are illustrated in several examples. One of them justifies the title of this paper. We conjecture that these mechanisms are deeply related to the very frequent emergence, in natural and artificial complex systems, of scale-free structures and to their connections with nonextensive statistical mechanics. Summarizing, we have shown that, for asymptotically scale-invariant systems, it is S_q with $q \neq 1$, and not S_{BG}, the entropy which matches standard, clausius-like, prescriptions of classical thermodynamics.

continuous variables ($N = 1, 2, 3$). In both cases, certain correlations that are scale-invariant in a suitable limit can create an intrinsically inhomogeneous occupation of phase space. Such systems are strongly reminiscent of the so called scale-free networks (24, 25), with their hierarchically structured hubs and spokes and their nearly forbidden regions.

Discrete Models

Some Basic Concepts. The most general probabilistic sets for N equal and distinguishable binary subsystems are given in Fig. 1 with

$$\sum_{n=0}^{N} \frac{N!}{(N-n)!}\, \pi_{N,n} = 1$$

$$(\pi_{N,n} \in [0, 1];\ N = 1, 2, 3, \ldots;\ n = 0, 1, \ldots, N). \quad [2]$$

Let us from now on call *Leibnitz rule* the following recursive relation:

$$\pi_{N,n} + \pi_{N,n+1} = \pi_{N-1,n}\ (n = 0, 1, \ldots, N-1;\ N = 2, 3, \ldots). \quad [3]$$

This relation guarantees what we refer to as *scale-invariance* (or *scale-freedom*) in this article. Indeed, it guarantees that, for any value of N, the associated *joint probabilities* $\{\pi_{N,n}\}$ produce *marginal probabilities* which coincide with $\{\pi_{N-1,n}\}$. Assuming $\pi_{10} + \pi_{11} =$

PHYSICS

Playing with additive duality $(q \to 2-q)$

and with multiplicative duality $(q \to 1/q)$

(*and using numerical results related to the q-generalized central limit theorem*)

we conjecture

$$q_{rel} + \frac{1}{q_{sen}} = 2 \qquad \text{and} \qquad q_{stat} + \frac{1}{q_{rel}} = 2$$

hence $$1 - q_{sen} = \frac{1 - q_{stat}}{3 - 2\, q_{stat}}$$

hence only one independent!

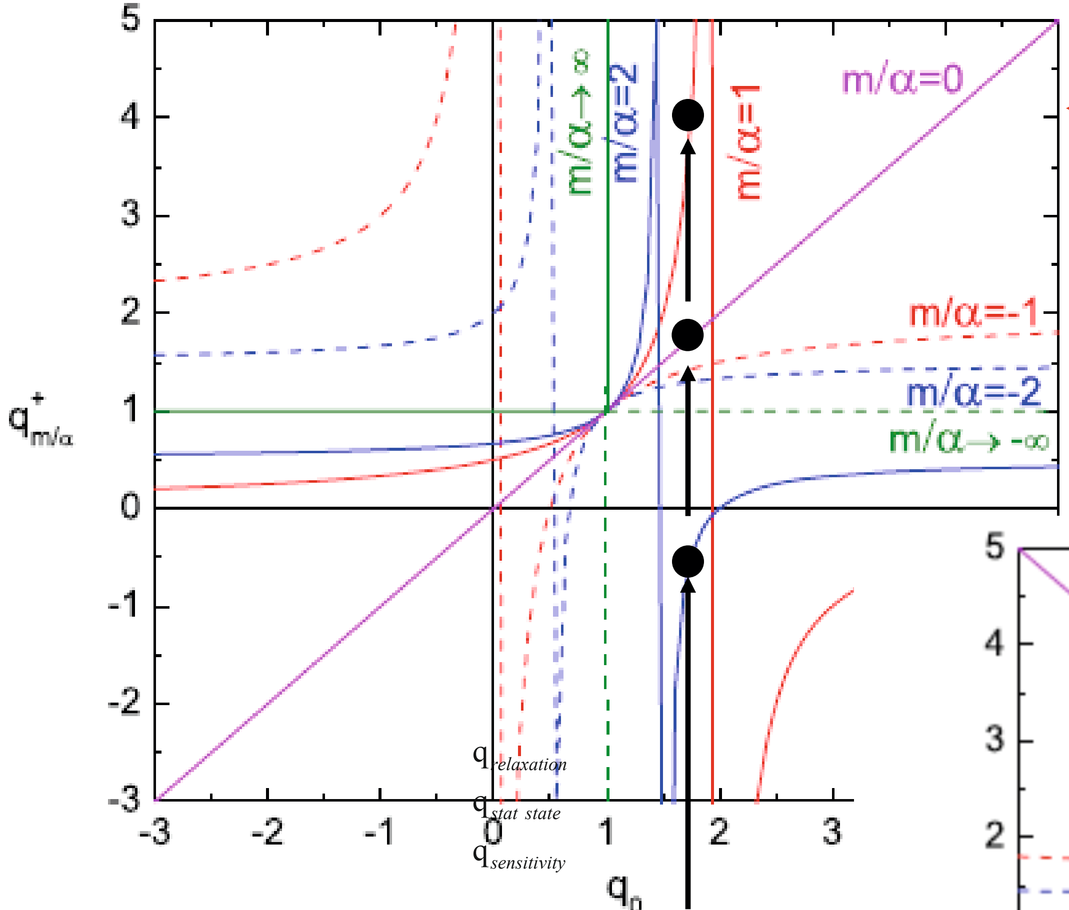
Burlaga and Vinas (NASA) most precise value of the q-triplet is

$$q_{stat} = 1.75 = 7/4$$

hence $q_{sen} = -0.5 = -1/2$ (*consistent with* $q_{sen} = -0.6 \pm 0.2$!)

and $q_{rel} = 4$ (*consistent with* $q_{rel} = 3.8 \pm 0.3$!)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

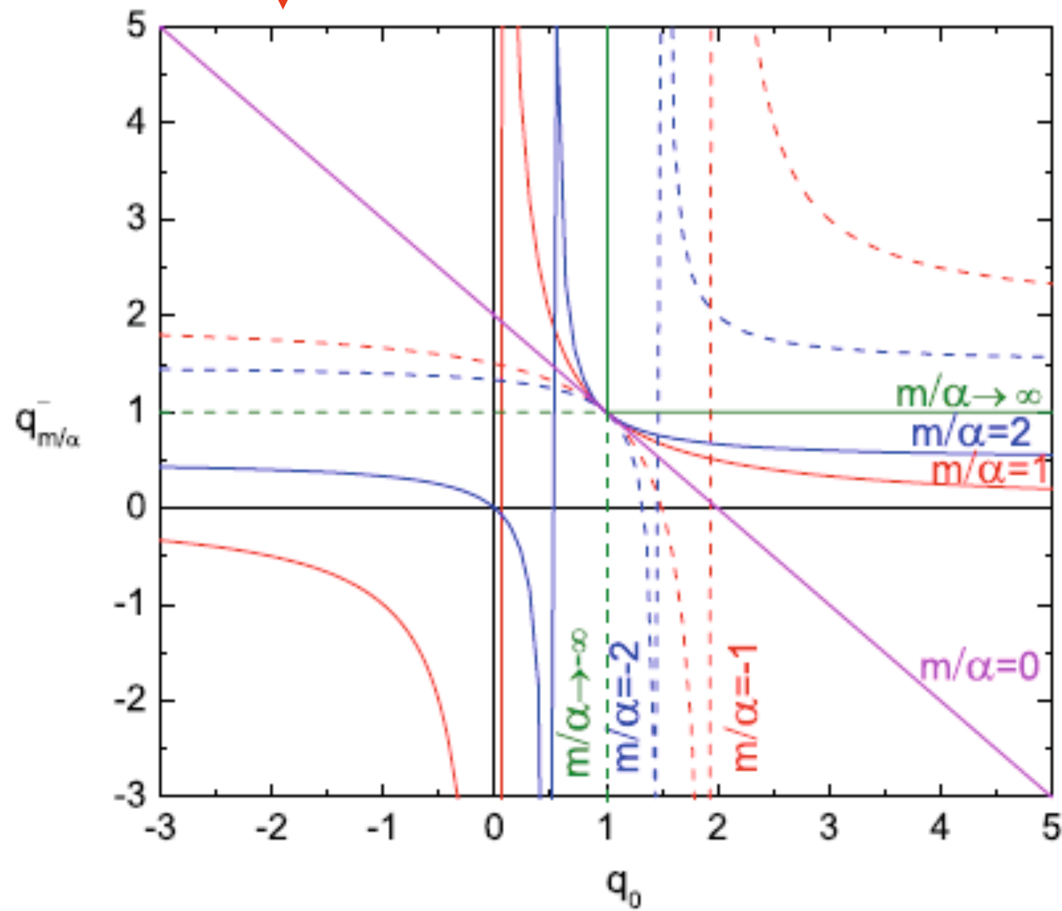


$$\frac{1}{1-q^{+}_{m/\alpha}} = \frac{1}{1-q_0} + \frac{m}{\alpha}$$

$$\frac{1}{1-q^{-}_{m/\alpha}} = \frac{1}{q_0-1} + \frac{m}{\alpha}$$

$$(0 < \alpha \leq 2;\ m = 0, \pm 1, \pm 2, \ldots)$$

solar wind
q-triplet
$q_{relaxation}$
$q_{stat\ state}$
$q_{sensitivity}$



C. T., Braz J Phys **39**, 337 (2009)

$$\varepsilon_{sen} \equiv 1 - q_{sen} = 1 - (-1/2) = 3/2$$

$$\varepsilon_{rel} \equiv 1 - q_{rel} = 1 - 4 = -3$$

$$\varepsilon_{stat} \equiv 1 - q_{stat} = 1 - 7/4 = -3/4$$

We verify

$$\varepsilon_{stat} = \frac{\varepsilon_{sen} + \varepsilon_{rel}}{2}$$ (arithmetic mean!)

$$\varepsilon_{sen} = \sqrt{\varepsilon_{stat} \ \varepsilon_{rel}}$$ (geometric mean!)

$$\varepsilon_{rel}^{-1} = \frac{\varepsilon_{sen}^{-1} + \varepsilon_{stat}^{-1}}{2}$$ (harmonic mean!)

N.O. Baella (2008)

EDGE OF CHAOS OF THE LOGISTIC MAP:

$$q\text{-triplet}\begin{cases} q_{sensitivity} = q_{entropy} = 0.2444877013412820661 98... \\ q_{relaxation} = 2.249784109... \\ q_{stationary\ state} = 1.65 \pm 0.05 \end{cases}$$

hence $q_{sens} < 1 < q_{stat} < q_{rel}$

CONJECTURE: [N.O. Baella (2010)] $\varepsilon \equiv 1 - q$

$$\varepsilon_{relaxation} + \varepsilon_{sensitivity} = \varepsilon_{sensitivity}\ \varepsilon_{stationary\ state}$$

hence

$$q_{stationary\ state} = \frac{q_{relaxation} - 1}{1 - q_{sensitivity}} = 1.65424...$$

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Radial velocities of open stellar clusters: A new solid constraint favouring Tsallis maximum entropy theory

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[b] *Departmento Cassiopée, UMR 6202, Observatoire de la Côte d'Azur, B.P. 4229, F-06304, Nice, Cedex 4, France*

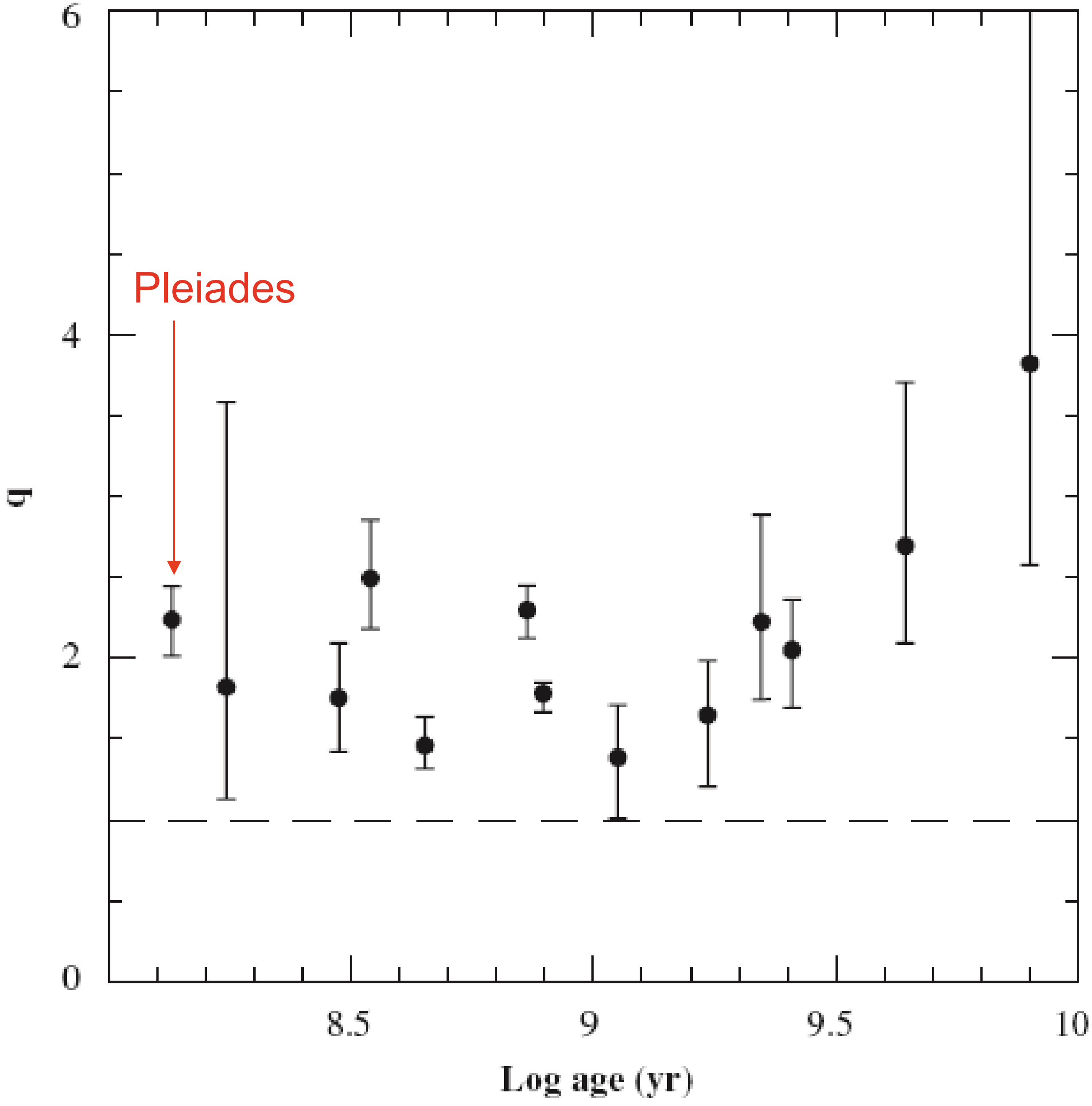

Fig. 5. Value of the fitted parameter q as a function of the cluster age.

THE ASTROPHYSICAL JOURNAL, 703:311–324, 2009 September 20

doi:10.1088/0004-637X/703

COMPRESSIBLE "TURBULENCE" OBSERVED IN THE HELIOSHEATH BY *VOYAGER 2*

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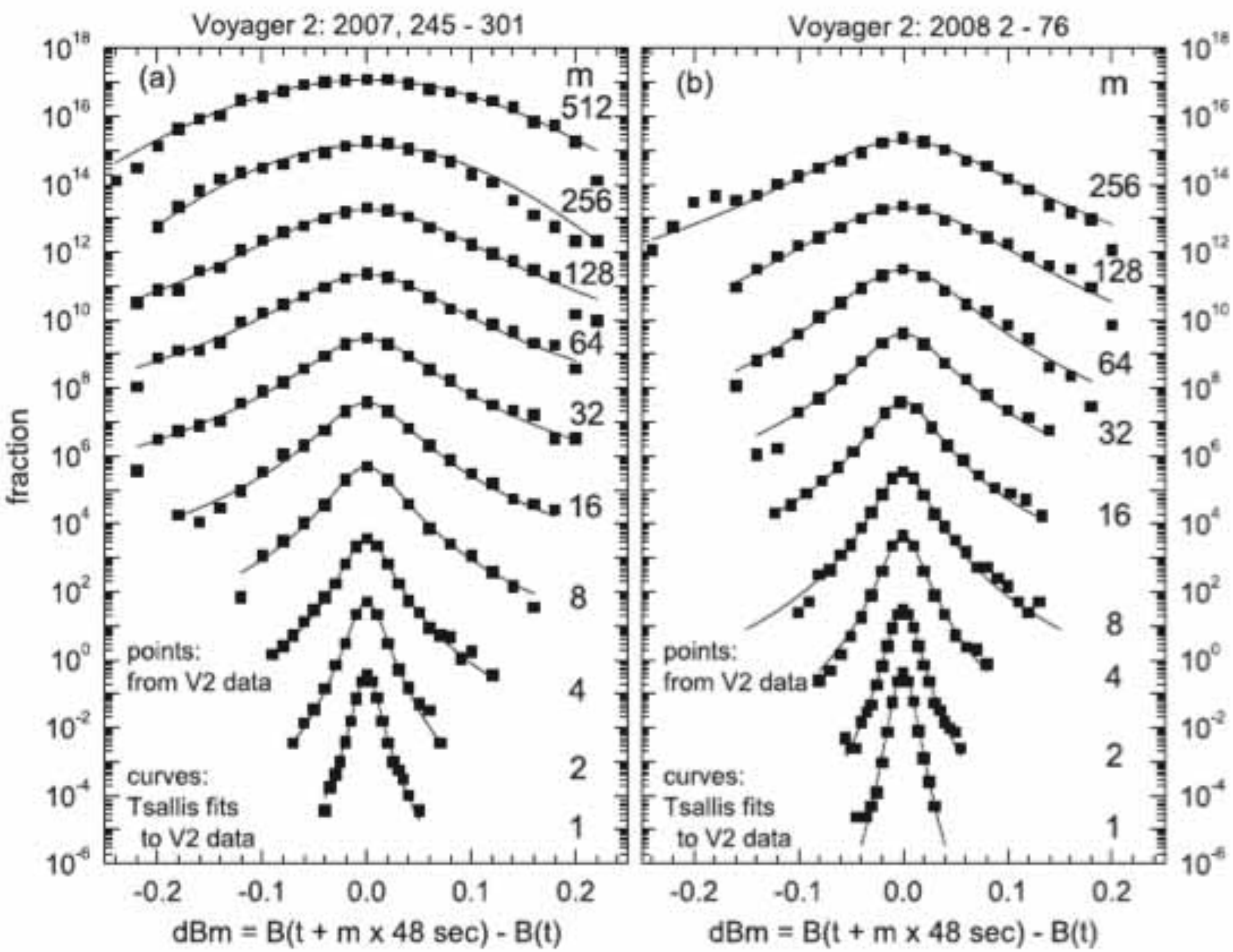
Received 2009 June 2; accepted 2009 July 22; published 2009 August 27



EDGE OF CHAOS OF THE LOGISTIC MAP:

(Using result in http://pi.lacim.uqam.ca/piDATA/feigenbaum.txt)

$$q = 1 - \frac{\ln 2}{\ln \alpha_F} =$$

0.2444877013412820661987704234046804052344469354900576736703650
986327749672766558665755156226857540706288349640382728306063600
193730331818964551341081277809792194386027083194490052465813521
503174534952074940448165460949087448334056723622466488083333072
142318987145872992681548496774607864821834569063370205946820461
899021675321457546117438305008496860408846969491704367478991506
016646491060217834827889993818382522554582338038113118031805448
236757944990397074395466146340815553168788535030113821491411266
246328940130370152354936571471269917921021622688833029675405780
6307068223688104320157903521237407354446029700060555250423142028
089193578811239731977974844235152456040926446709579570304658614
129566479666687743683240492022757393004750895311855179558720483
992696896827555852445024436526825609423780128033094877954403542
524859043379761802711830004573585550738941136758784400629135630
421674541694092135698603207859088199859359007319336801069967496
707904456092418632112054130547393985795544410347612222592136846
219346009360… (1018 meaningful digits)

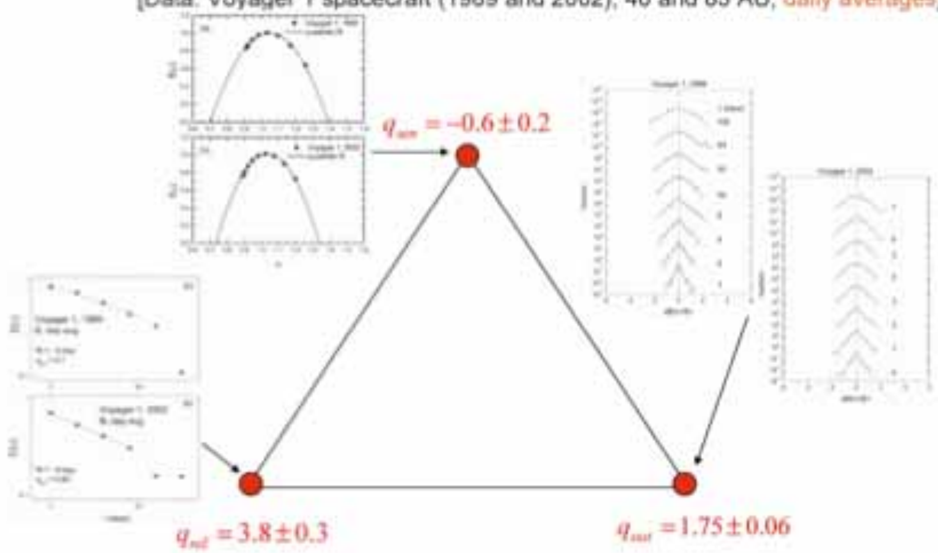
IHY 2007: VOYAGER 1: Fundamental Physics

The atmosphere of the Sun beyond a few solar radii, known as HELIOSPHERE, is fully ionized plasma expanding at supersonic speeds, carrying solar magnetic fields with it. This solar wind is a driven non-linear non-equilibrium system. The Sun injects matter, momentum, energy, and magnetic fields into the heliosphere in a highly variable way. Voyager 1 observed magnetic field strength variations in the solar wind near 40 AU during 1989 and near 85 AU during 2002. Tsallis' non-extensive statistical mechanics, a generalization of Boltzmann-Gibbs statistical mechanics, allows a physical explanation of these magnetic field strength variations in terms of departure from thermodynamic equilibrium in an unique way:

SOLAR WIND: Magnetic Field Strength

L.F. Burlaga and A. F.-Vinas (2005) / NASA Goddard Space Flight Center

[Data: Voyager 1 spacecraft (1989 and 2002); 40 and 85 AU; daily averages]



Nonextensive statistical mechanics and thermodynamics

C. T.

Possible generalization of Boltzmann-Gibbs statistics

J Stat Phys **52**, 479 (1988)

E.M.F. Curado and C. T.

Generalized statistical mechanics: connection with thermodynamics

J Phys A **24**, L69 (1991)

[Corrigenda: **24**, 3187 (1991) and **25**, 1019 (1992)]

C. T., R.S. Mendes and A.R. Plastino

The role of constraints within generalized nonextensive statistics

Physica A **261**, 534 (1998)

NONEXTENSIVE STATISTICAL MECHANICS AND THERMODYNAMICS (CANONICAL ENSEMBLE):

Extremization of the functional

$$S_q[p_i] \equiv k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$$

with the constraints

$$\sum_{i=1}^{W} p_i = 1 \quad \textit{and} \quad \frac{\sum_{i=1}^{W} p_i^q E_i}{\sum_{i=1}^{W} p_i^q} = U_q$$

yields

$$p_i = \frac{e_q^{-\beta_q (E_i - U_q)}}{\mathbf{Z}_q}$$

with $\beta_q \equiv \dfrac{\beta}{\sum_{i=1}^{W} p_i^q}$, $\beta \equiv$ *energy Lagrange parameter*, *and* $\mathbf{Z}_q \equiv \sum_{i=1}^{W} e_q^{-\beta_q (E_i - U_q)}$

We can rewrite $$p_i = \frac{e_q^{-\beta_q' E_i}}{Z_q'}$$

with $$\beta_q' \equiv \frac{\beta_q}{1+(1-q)\beta_q U_q}\ , \quad \textit{and} \quad Z_q' \equiv \sum_{i=1}^{W} e_q^{-\beta_q' E_i}$$

And we can prove

$$(i) \quad \frac{1}{T} = \frac{\partial S_q}{\partial U_q} \quad \textit{with} \quad T \equiv \frac{1}{k\beta}$$

$$(ii) \quad F_q \equiv U_q - TS_q = -\frac{1}{\beta}\ln_q Z_q \quad \textit{where} \quad \ln_q Z_q = \ln_q \mathbf{Z}_q - \beta U_q$$

$$(iii) \quad U_q = -\frac{\partial}{\partial \beta}\ln_q Z_q$$

$$(iv) \quad C_q \equiv T\frac{\partial S_q}{\partial T} = \frac{\partial U_q}{\partial T} = -T\frac{\partial^2 F_q}{\partial T^2}$$

(*i.e., the Legendre structure of Thermodynamics is q-invariant!*)

Tunable Tsallis Distributions in Dissipative Optical Lattices

P. Douglas, S. Bergamini, and F. Renzoni

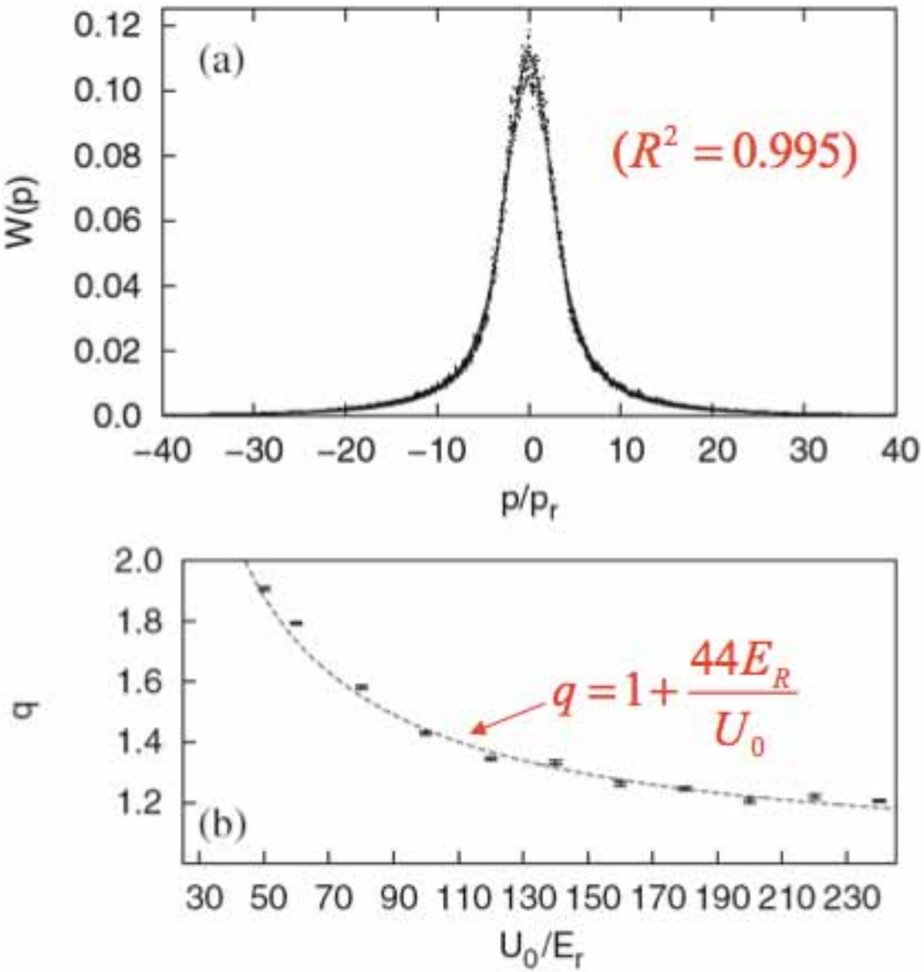
Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom

(Received 10 January 2006; published 24 March 2006)

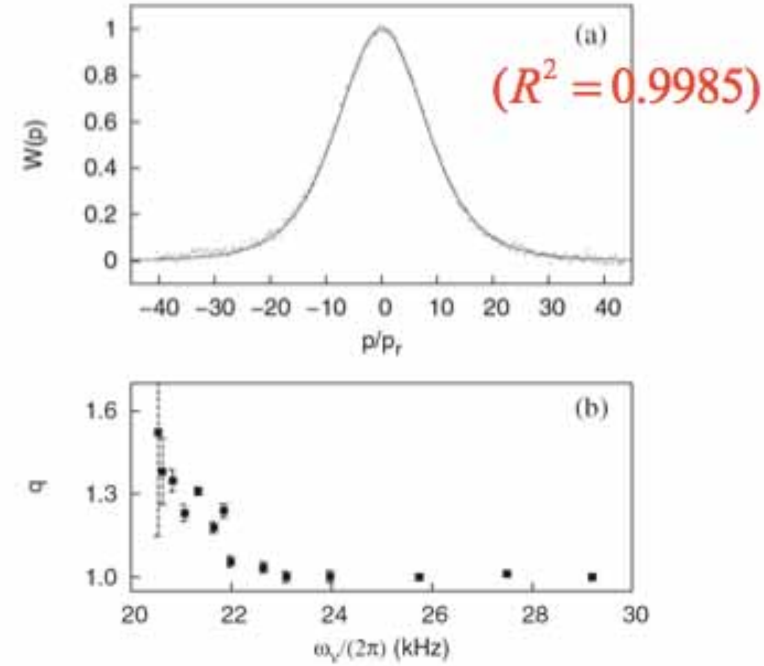
We demonstrated experimentally that the momentum distribution of cold atoms in dissipative optical lattices is a Tsallis distribution. The parameters of the distribution can be continuously varied by changing the parameters of the optical potential. In particular, by changing the depth of the optical lattice, it is possible to change the momentum distribution from Gaussian, at deep potentials, to a power-law tail distribution at shallow optical potentials.

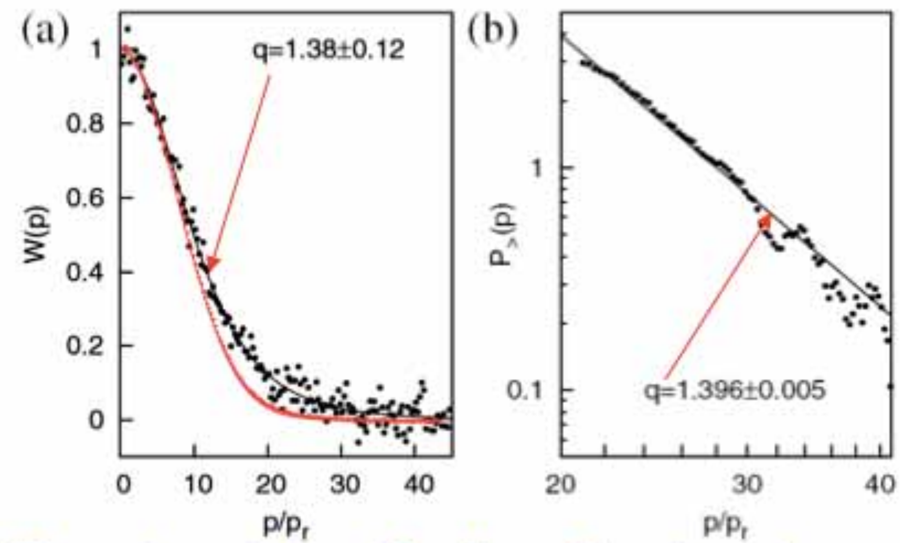
Experimental and computational verifications

by P. Douglas, S. Bergamini and F. Renzoni, Phys Rev Lett **96**, 110601 (2006)



(Computational verification: quantum Monte Carlo simulations)





(Experimental verification: Cs atoms)