NONADDITIVE ENTROPIES:

FROM EINSTEIN TO SPACE WEATHER

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and

Santa Fe Institute, New Mexico, USA

Fukuoka, March 2015

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ **equiprobability**	$\forall p_i \; (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1 \right)$
BG entropy (*q* =*1*)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (*q real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

C.T., J Stat Phys **52**, 479 (1988)

DEFINITIONS: *q-logarithm*: $\ln_q x \equiv \frac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q-exponential: $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k\ \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k\ \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

11. *Theorie der Opaleszenz von homogenen Flüssigkeiten und Flüssigkeitsgemischen in der Nähe des kritischen Zustandes;*

von A. Einstein.

$$S = \frac{R}{N} \lg W + \text{konst.} \tag{1}$$

Daß die zwischen S und W in Gleichung (1) gegebene Beziehung die einzig mögliche ist, kann bekanntlich aus dem Satze abgeleitet werden, daß die Entropie eines aus Teilsystemen bestehenden Gesamtsystems gleich ist der Summe der Entropien der Teilsysteme.

The relation between *S* and *W* given in Eq. (1) is the only reasonable given the proposition that the entropy of a system consisting of subsystems is equal to the sum of entropies of the subsystems.

(Free translation by Tobias Micklitz)

BOLTZMANN-GIBBS ENTROPY IS SUFFICIENT BUT NOT NECESSARY FOR THE LIKELIHOOD FACTORIZATION REQUIRED BY EINSTEIN

C. T. and H.J. Haubold (2014), 1407.6052 [cond-mat.stat-mech]

Einstein 1910 (reversal of Boltzmann formula):

For any two independent systems A and B,

the likelihood function should satisfy

$$\Omega(A+B)=\Omega(A)\,\Omega(B) \quad \text{(Einstein principle)}$$

$q=1:$ $\quad S_{BG}=k_B \ln W$ hence $\Omega(\{p_i\}) \propto e^{S_{BG}(\{p_i\})/k_B}$ hence

$$\Omega(A+B) \propto e^{S_{BG}(A+B)/k_B} = e^{S_{BG}(A)/k_B + S_{BG}(B)/k_B} = e^{S_{BG}(A)/k_B} e^{S_{BG}(B)/k_B} \propto \Omega(A)\,\Omega(B)$$

OK!

$\forall q:$ $\quad S_q=k_B \ln_q W$ hence $\Omega(\{p_i\}) \propto e_q^{S_q(\{p_i\})/k_B}$ hence

$$\Omega(A+B) \propto \mathrm{e}_q^{S_q(A+B)/k_B} = \mathrm{e}_q^{S_q(A)/k_B + S_q(B)/k_B + (1-q)[S_q(A)/k_B][S_q(B)/k_B]}$$

$$= e_q^{S_q(A)/k_B} e_q^{S_q(B)/k_B} \propto \Omega(A)\,\Omega(B)$$

OK $\forall q$!

A Letters Journal Exploring the Frontiers of Physics

October 2009

EPL, **88** (2009) 19001
doi: 10.1209/0295-5075/88/19001

www.epljournal.org

Nonextensivity in the solar magnetic activity during the increasing phase of solar cycle 23

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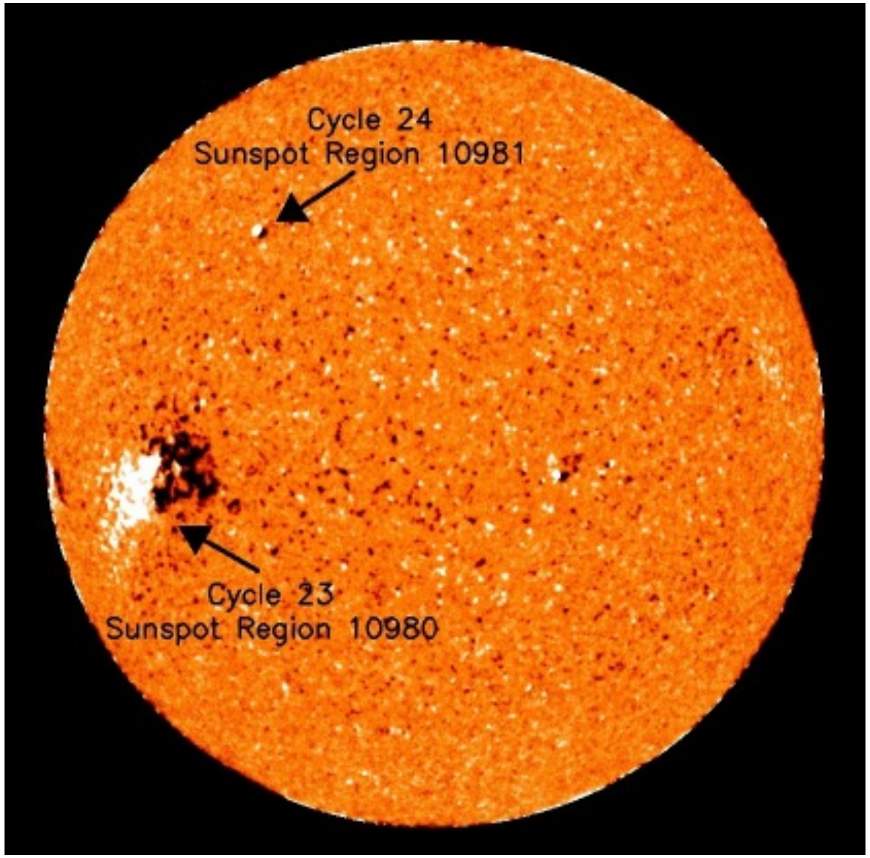

The Astrophysical Journal, 703:311–324, 2009 September 20 doi:10.1088/0004-637X/703

COMPRESSIBLE "TURBULENCE" OBSERVED IN THE HELIOSHEATH BY *VOYAGER 2*

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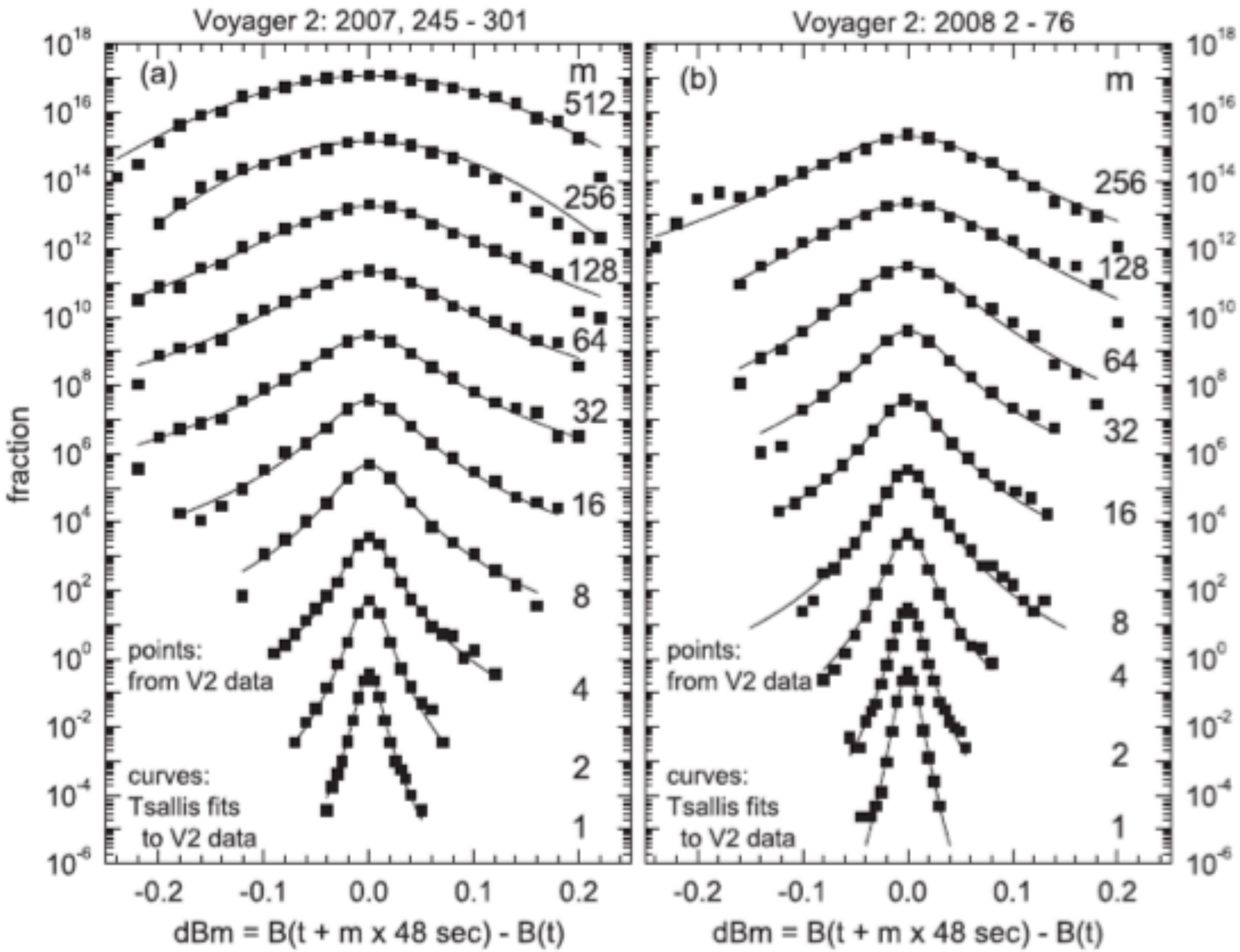
[2] Institute for Astrophysics and Computational Sciences, Catholic University of America, Washington DC 20064, USA; nfnudel@yahoo.com
Received 2009 June 2; accepted 2009 July 22; published 2009 August 27

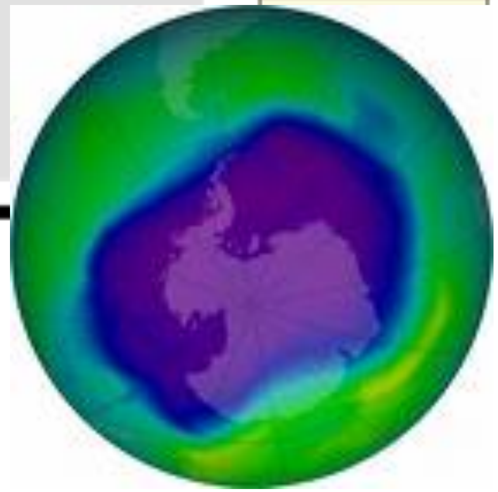


Physica A 389 (2010) 1829–1833

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa



Tsallis' *q*-triplet and the ozone layer

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ARTICLE INFO

Article history:
Received 23 September 2009
Received in revised form 24 November 2009
Available online 22 December 2009

ABSTRACT

Tsallis' *q*-triplet [C. Tsallis, Dynamical scenario for nonextensive statistical mechanics, Physica A 340 (2004) 1–10] is the best empirical quantifier of nonextensivity. Here we study it with reference to an experimental time-series related to the daily depth-values of the stratospheric ozone layer. Pertinent data are expressed in Dobson units and range from 1978 to 2005. After the evaluation of the three associated Tsallis' indices one concludes that nonextensivity is clearly a characteristic of the system under scrutiny.

Physica A 391 (2012) 3069–3080

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Tsallis statistics and magnetospheric self-organization

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ARTICLE INFO

Article history:
Received 24 October 2011
Received in revised form 9 January 2012
Available online 23 January 2012

Keywords:
Tsallis non-extensive statistics
Non-equilibrium phase transition
Intermittent turbulence
Self-organized criticality
Low dimensional chaos
Magnetosphere
Superstorm

ABSTRACT

In this study we use Tsallis non-extensive statistics for a new understanding the magnetospheric dynamics and the magnetospheric self-organization during quiet and intensive superstorm periods. The q_{sens}, q_{stat}, and q_{rel} indices set known as the Tsallis q-triplet was estimated during both quiet and strongly active periods, as well as the correlation dimensions and Lyapunov exponents spectrum for magnetospheric bulk plasma flows data. The results obtained by our analysis clearly indicate the magnetospheric phase transition process from a high-dimensional quiet SOC state to a low-dimensional global chaotic state when superstorm events are developed. During such a phase transition process the non-extensive statistical character of the magnetospheric plasma is strengthened as the values of the q-triplet indices changes obtaining higher values than their values during the quiet periods.

IC-MSQUARE 2014 IOP Publishing
Journal of Physics: Conference Series **574** (2015) 012128 doi:10.1088/1742-6596/574/1/012128

Application of nonlinear methods to the study of ionospheric plasma

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Abstract. Most of the processes taking place in the auroral region of Earth's ionosphere are reflected in a variety of dynamic forms of the aurora borealis. In order to study these processes it is necessary to consider temporary and spatial variations of the characteristics of ionospheric plasma. Most traditional methods of classical physics are applicable mainly for stationary or quasi-stationary phenomena, but dynamic regimes, transients, fluctuations, self-similar scaling could be considered using the methods of nonlinear dynamics. Special interest is the development of the methods for describing the spatial structure and the temporal dynamics of auroral ionosphere based on the ideas of percolation theory and fractal geometry. The fractal characteristics (the Hausdorff fractal dimension and the index of connectivity) of Hall and Pedersen conductivities are used to the description of fractal patterns in the ionosphere. To obtain the self-consistent estimates of the parameters the Hausdorff fractal dimension and the index of connectivity in the auroral zone, an additional relation describing universal behavior of the fractal geometry of percolation at the critical threshold is applied. Also, it is shown that Tsallis statistics can be used to study auroral ionosphere

3. q-statistics

We demonstrate for the first time that q-statistics [2] can be used to study the Earth's auroral region. In order to show that q-statistics may apply for study of auroral ionosphere, we use pulsating aurora event on 2011-12-03 from 22:00UT which were observed by Apatity all-sky camera [7]. The obtained data (where background was subtracted and bright stars were deleted) were divided into ten time intervals and for each interval we calculated the value of parameter q. The parameter q is estimated using maximum likelihood. Specifically the maximum likelihood estimates (MLEs) have been determined from a solution of the stationary points of the log likelihood function. Parameter q indicates a departure from Gaussian distribution. In the limit $q \to 1$ we have usual Gaussian statistics.

Our study shows that when bright auroras are observed, there is a strong deviation from unity of the parameter q. Otherwise, the value of q tends to unity, that is, to Gaussian statistics. This is due to the fact that during auroral glow ionospheric processes are substantially non-Gaussian and strong intermittency occurs. Also, it is established a good correlation between the change in values of q and flatness. While aurora glows are observed, flatness and parameter q grow and vice versa. In fact, it is seen that the Tsallis statistics can be used to study of non-Gaussian process, intermittency, along with flatness and/or probability density function (PDF).

The standard map:
From Boltzmann-Gibbs statistics to Tsallis statistics

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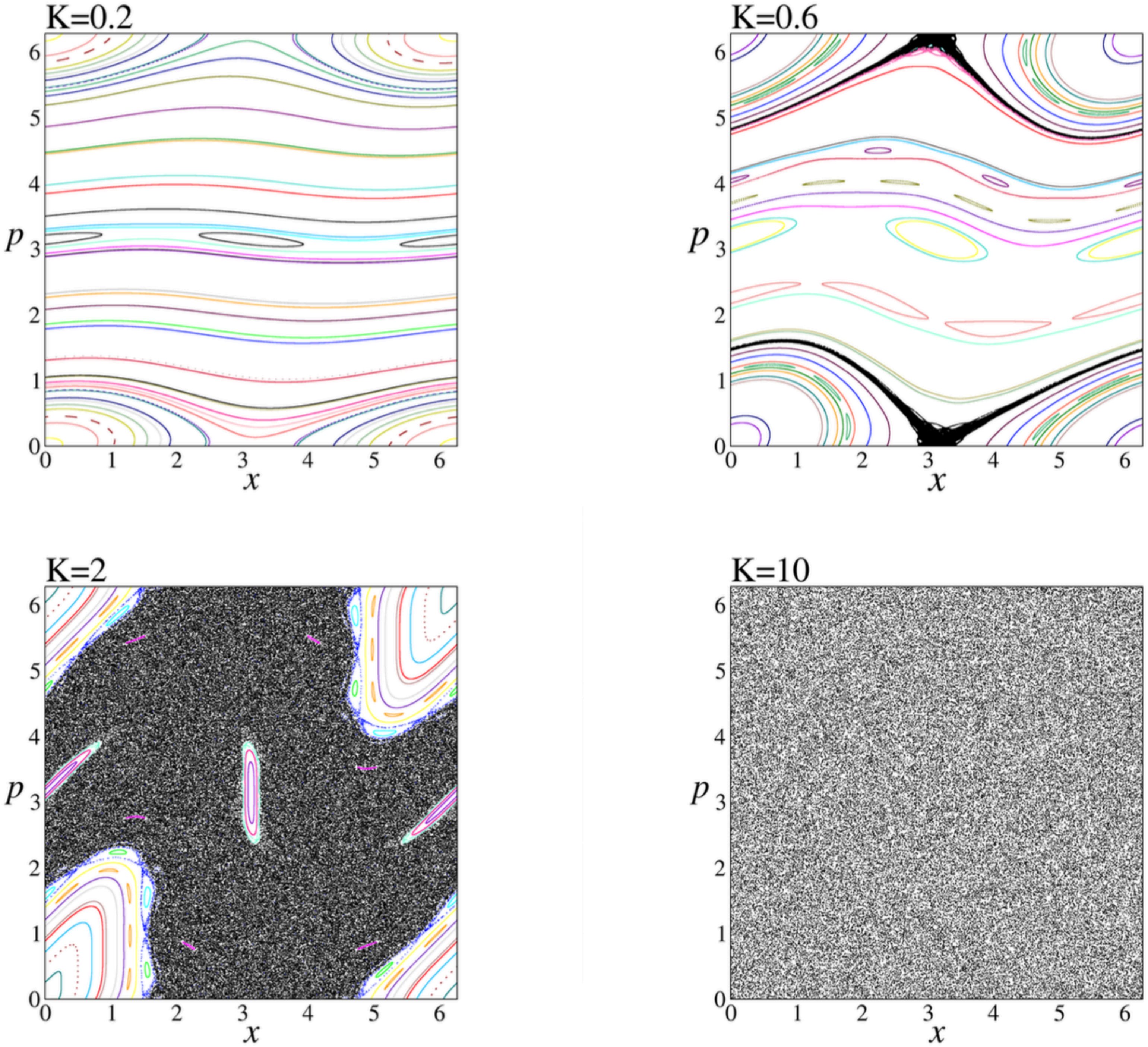
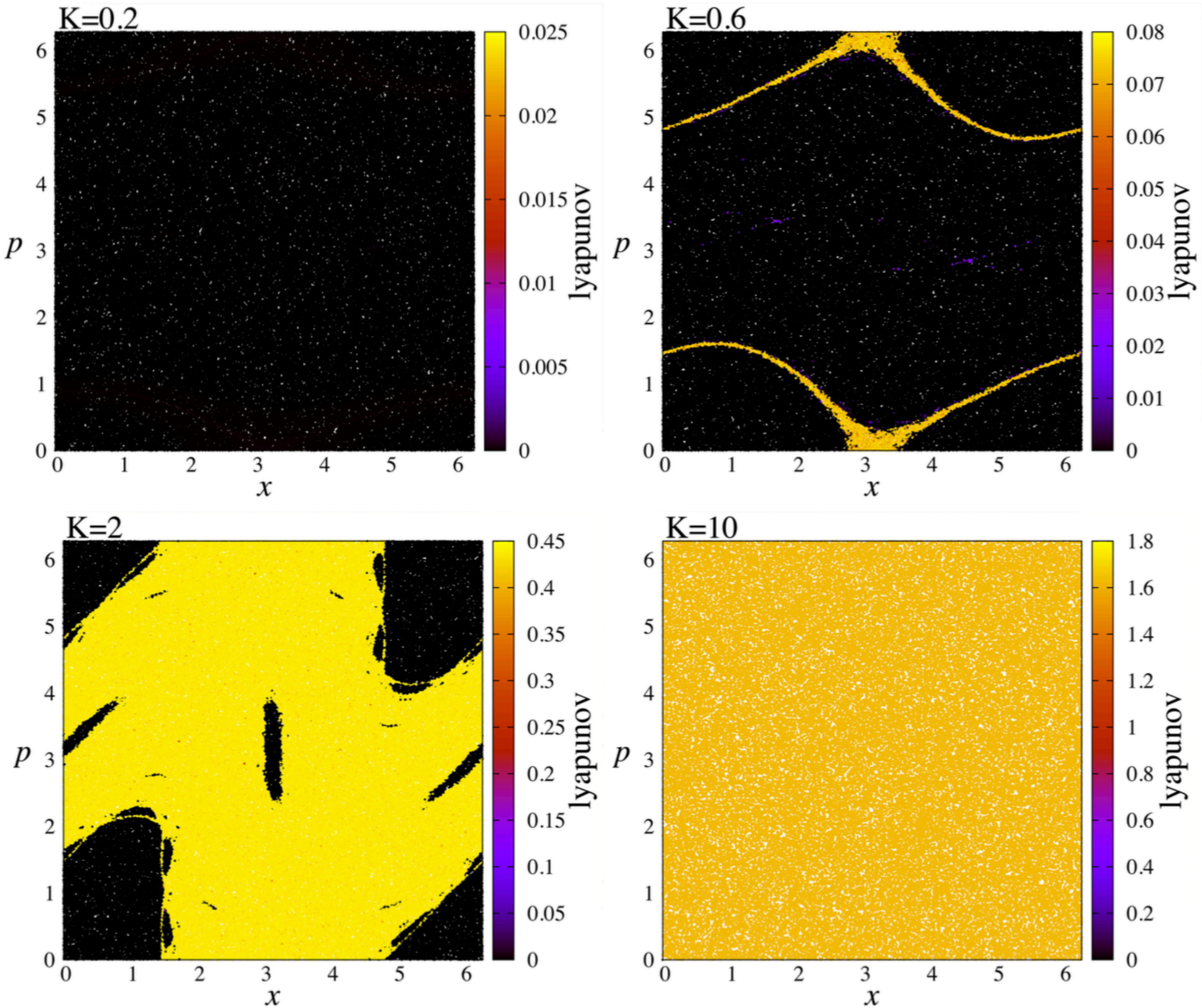
(Dated: January 16, 2015)

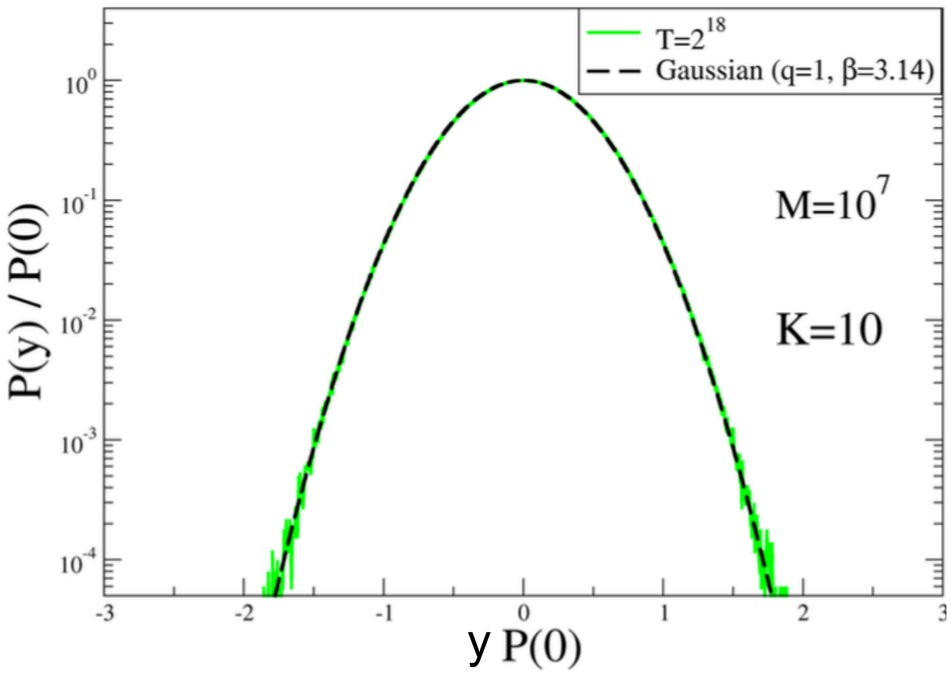
1501.02459 [cond-mat.stat-mech]

$$p_{i+1} = p_i - K \sin x_i$$
$$x_{i+1} = x_i + p_{i+1} \qquad (i = 0,1,2,\ldots)$$

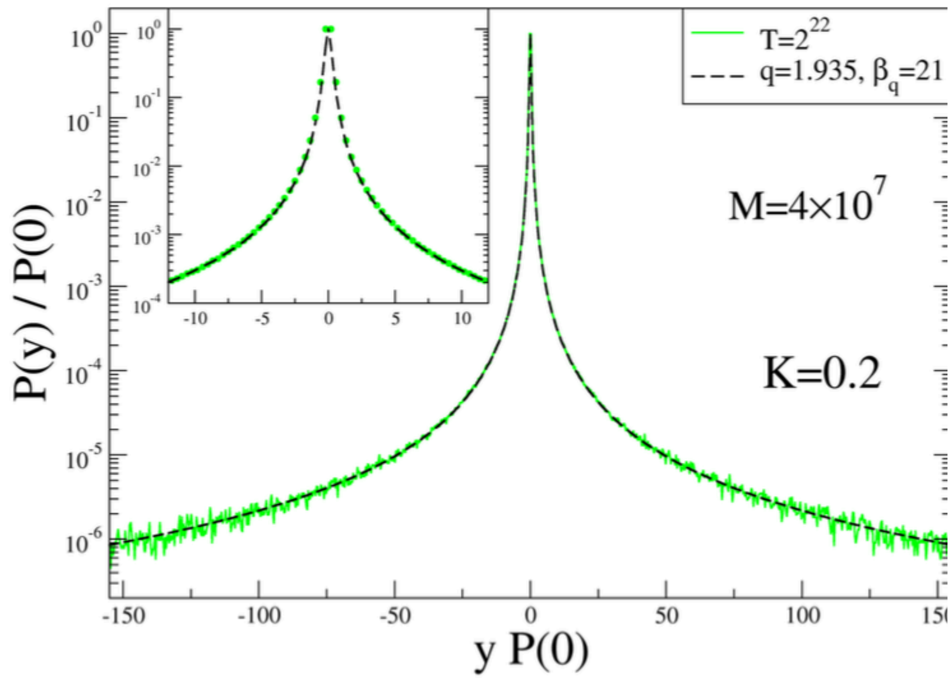
Particle confinement in magnetic traps, particle dynamics in accelerators, comet dynamics, ionization of Rydberg atoms, electron magneto-transport


K=0.2
K=0.6
K=2
K=10
p
x
0 1 2 3 4 5 6

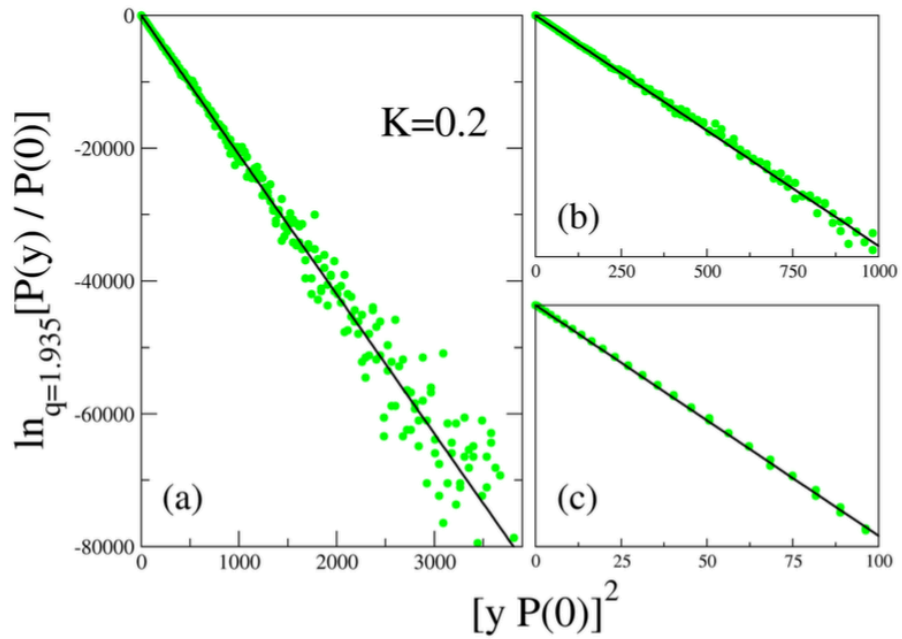

K=0.2
K=0.6
K=2
K=10
p
x
lyapunov
0.025
0.02
0.015
0.01
0.005
0.08
0.07
0.06
0.05
0.04
0.03
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6



T=2^18
Gaussian (q=1, β=3.14)
M=10^7
K=10
P(y) / P(0)
y P(0)



T=2^22
q=1.935, β_q=21
M=4×10^7
K=0.2
P(y) / P(0)
y P(0)



K=0.2
(a)
(b)
(c)
ln_q=1.935[P(y) / P(0)]
[y P(0)]^2

WHAT DO WE GAIN FROM KNOWING THE VALUES OF q?

- We now know how to calculate specific heats, equations of states, magnetic and electric susceptibilities, … and all the thermostatistical jazz!

- We now know how quickly neighboring initial conditions get apart with time.

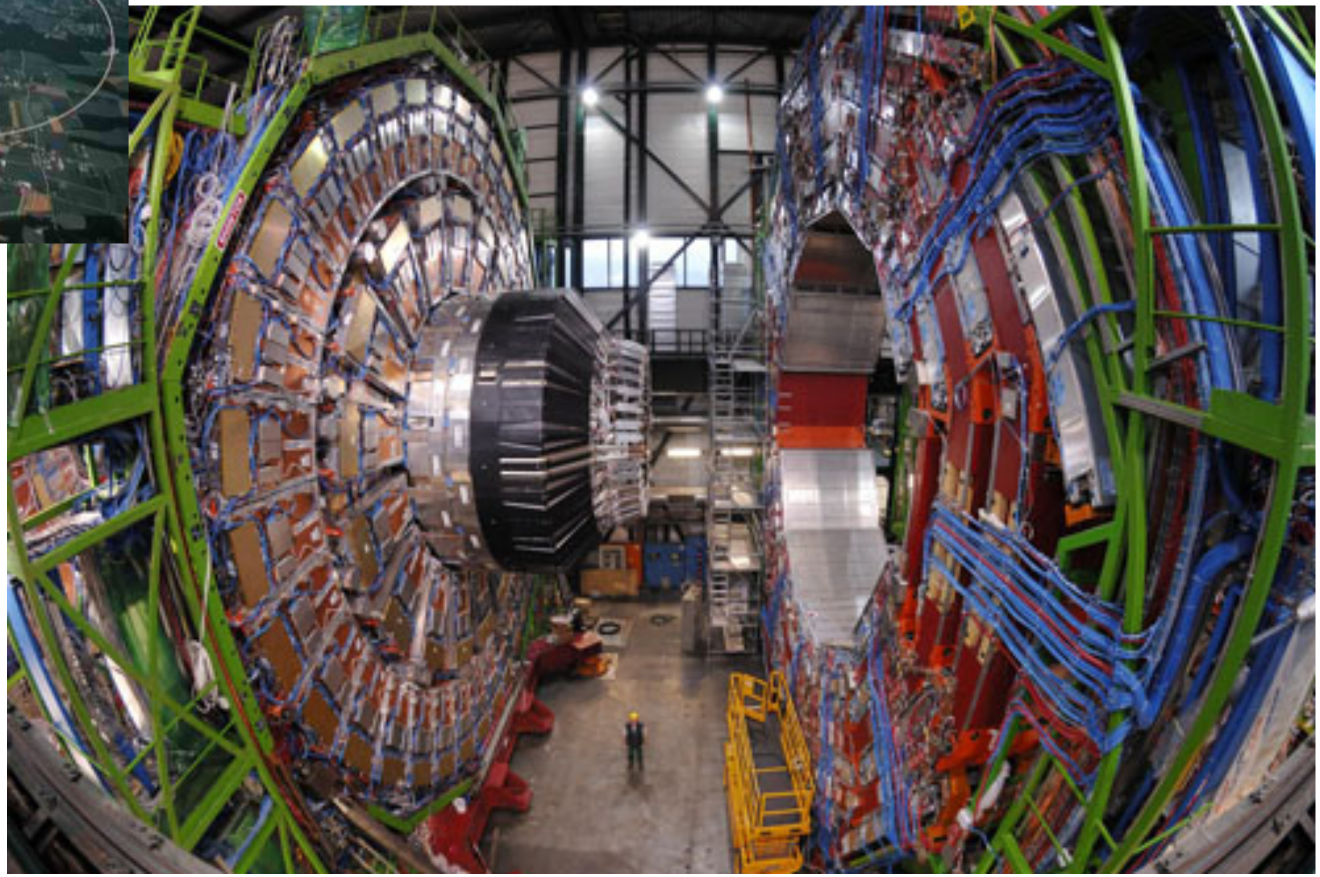
- We now know what physical mechanisms are possible:

  Long-range-interacting many-body classical Hamiltonians (e.g., generalized Fermi-Pasta-Ulam system);
  Subsystem of strongly quantum-entangled systems (e.g., part of a transverse-field Ising model at zero temperature critical point);
  Many-body systems with strongly dissipative mechanisms (e.g., overdamped motion of repulsively interacting vortices in type II superconductors);
  Dissipative maps at the edge of chaos (e.g., the logistic map at the Feigenbaum point);
  Conservative maps whose maximal Lyapunov exponent is nearly zero (e.g., standard map).

LHC (Large Hadron Collider)

CMS (Compact Muon Solenoid) detector

~ 2500 scientists/engineers from 183 institutions of 38 countries



PHYSICAL REVIEW D **87,** 114007 (2013)

Tsallis fits to p_T spectra and multiple hard scattering in pp collisions at the LHC

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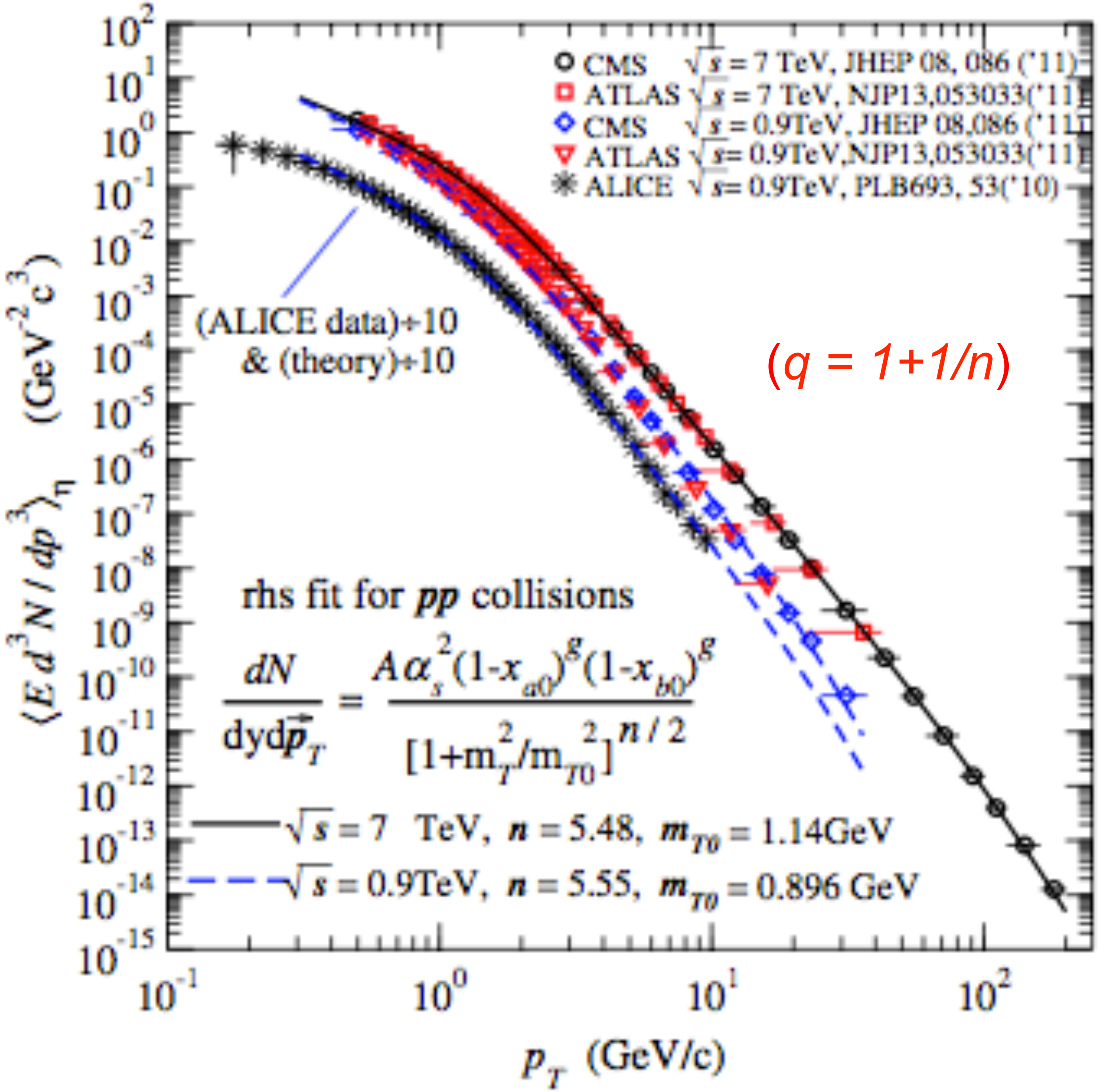
(Received 12 May 2013; published 5 June 2013)

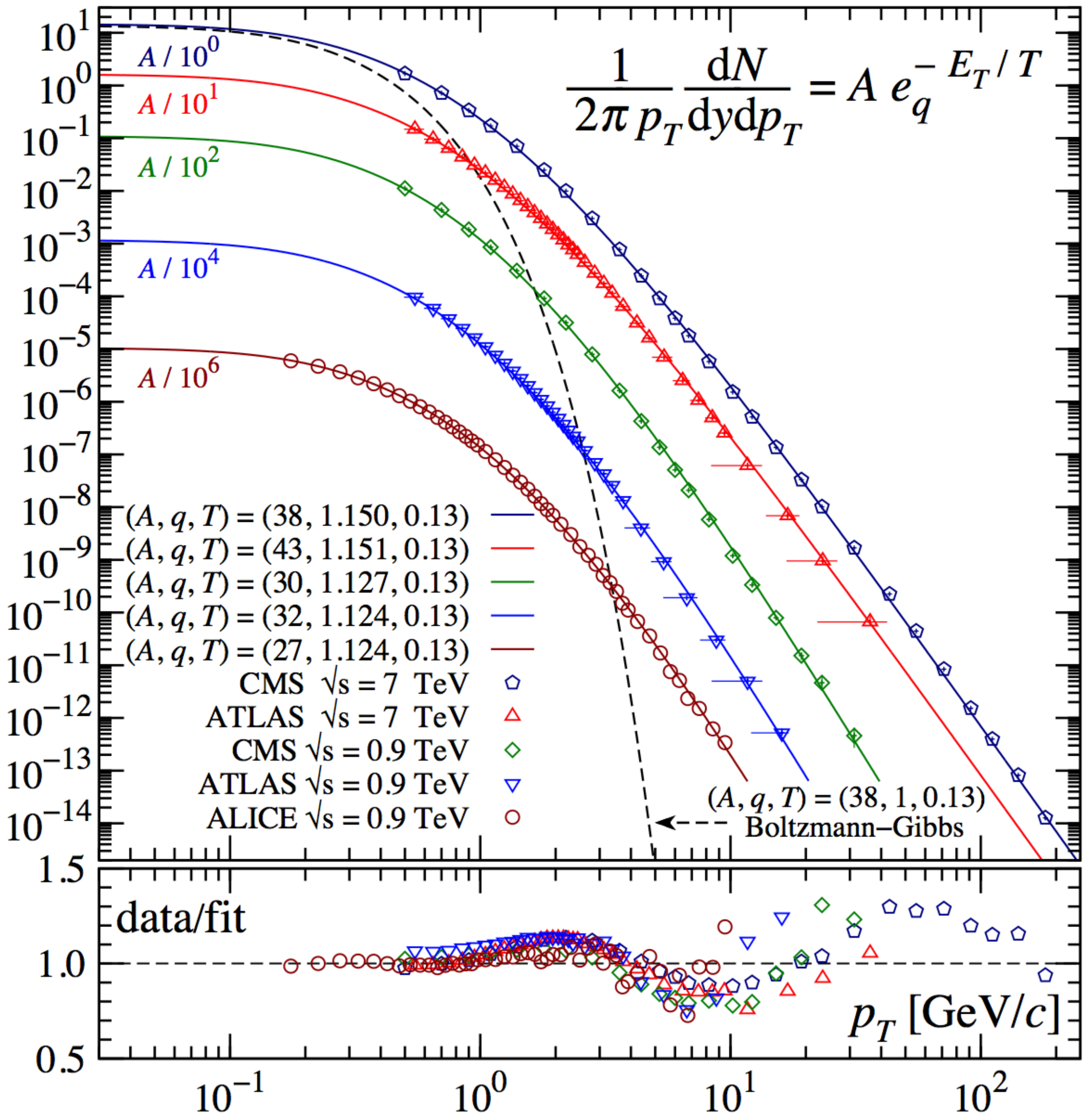
Phenomenological Tsallis fits to the CMS, ATLAS, and ALICE transverse momentum spectra of hadrons for pp collisions at LHC were recently found to extend over a large range of the transverse momentum. We investigate whether the few degrees of freedom in the Tsallis parametrization may arise from the relativistic parton-parton hard-scattering and related processes. The effects of the multiple hard-scattering and parton showering processes on the power law are discussed. We find empirically that whereas the transverse spectra of both hadrons and jets exhibit power-law behavior of $1/p_T^n$ at high p_T, the power indices n for hadrons are systematically greater than those for jets, for which $n \sim 4$–5.



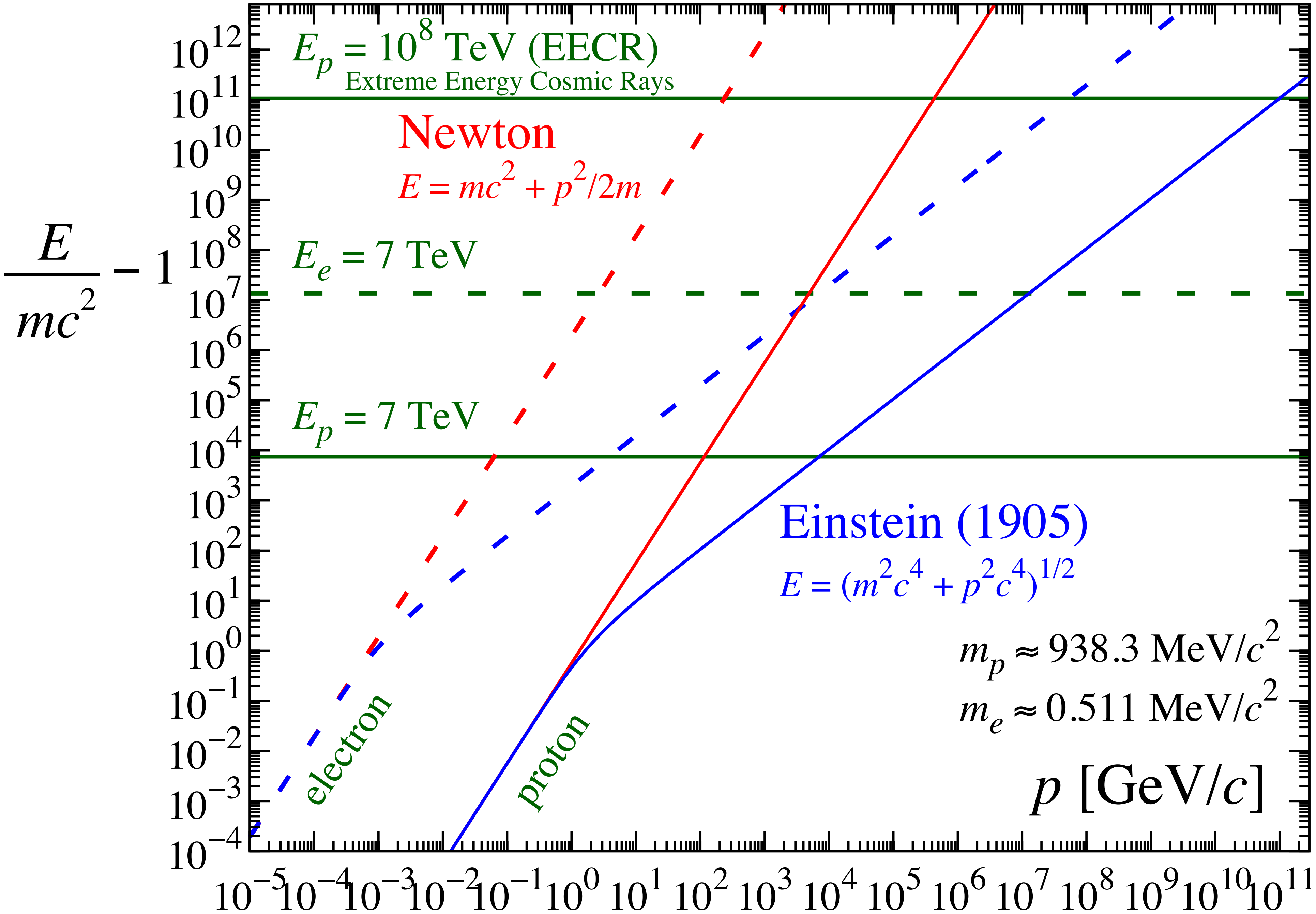
CMS √s = 7 TeV, JHEP 08, 086 ('11)
ATLAS √s = 7 TeV, NJP13,053033('11)
CMS √s = 0.9TeV, JHEP 08,086 ('11)
ATLAS √s= 0.9TeV,NJP13,053033('11)
ALICE √s= 0.9TeV, PLB693, 53('10)
(ALICE data)÷10
& (theory)÷10
rhs fit for pp collisions
$$\frac{dN}{dyd\vec{p}_T} = \frac{A\alpha_s^2(1-x_{a0})^g(1-x_{b0})^g}{[1+m_T^2/m_{T0}^2]^{n/2}}$$
√s = 7 TeV, n = 5.48, m_T0 = 1.14GeV
√s = 0.9TeV, n = 5.55, m_T0 = 0.896 GeV
⟨E d³N / dp³⟩_η (GeV⁻² c³)
p_T (GeV/c)
(q = 1+1/n)

SIMPLE APROACH: TWO-DIMENSIONAL RELATIVISTIC FREE PARTICLE

L.J.L. Cirto, C. T., C.Y. Wong and G. Wilk, 1409.3278 [hep-ph]



$$E_T = \sqrt{m^2c^4 + p_T^2c^2}$$



$E_p = 10^8$ TeV (EECR)
Extreme Energy Cosmic Rays
Newton
$E = mc^2 + p^2/2m$
$E_e = 7$ TeV
$E_p = 7$ TeV
Einstein (1905)
$E = (m^2c^4 + p^2c^4)^{1/2}$
$m_p \approx 938.3$ MeV/c^2
$m_e \approx 0.511$ MeV/c^2
electron
proton
$\frac{E}{mc^2} - 1$
p [GeV/c]
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12}
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^{10} 10^{11}

Entropy **2015**, *17*, 384-400; doi:10.3390/e17010384

OPEN ACCESS
entropy
ISSN 1099-4300
www.mdpi.com/journal/entropy

Article

Tsallis Distribution Decorated with Log-Periodic Oscillation

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[2] Institute of Physics, Jan Kochanowski University, Świętokrzyska 15, 25-406 Kielce, Poland; E-Mail:zbigniew.wlodarczyk@ujk.edu.pl

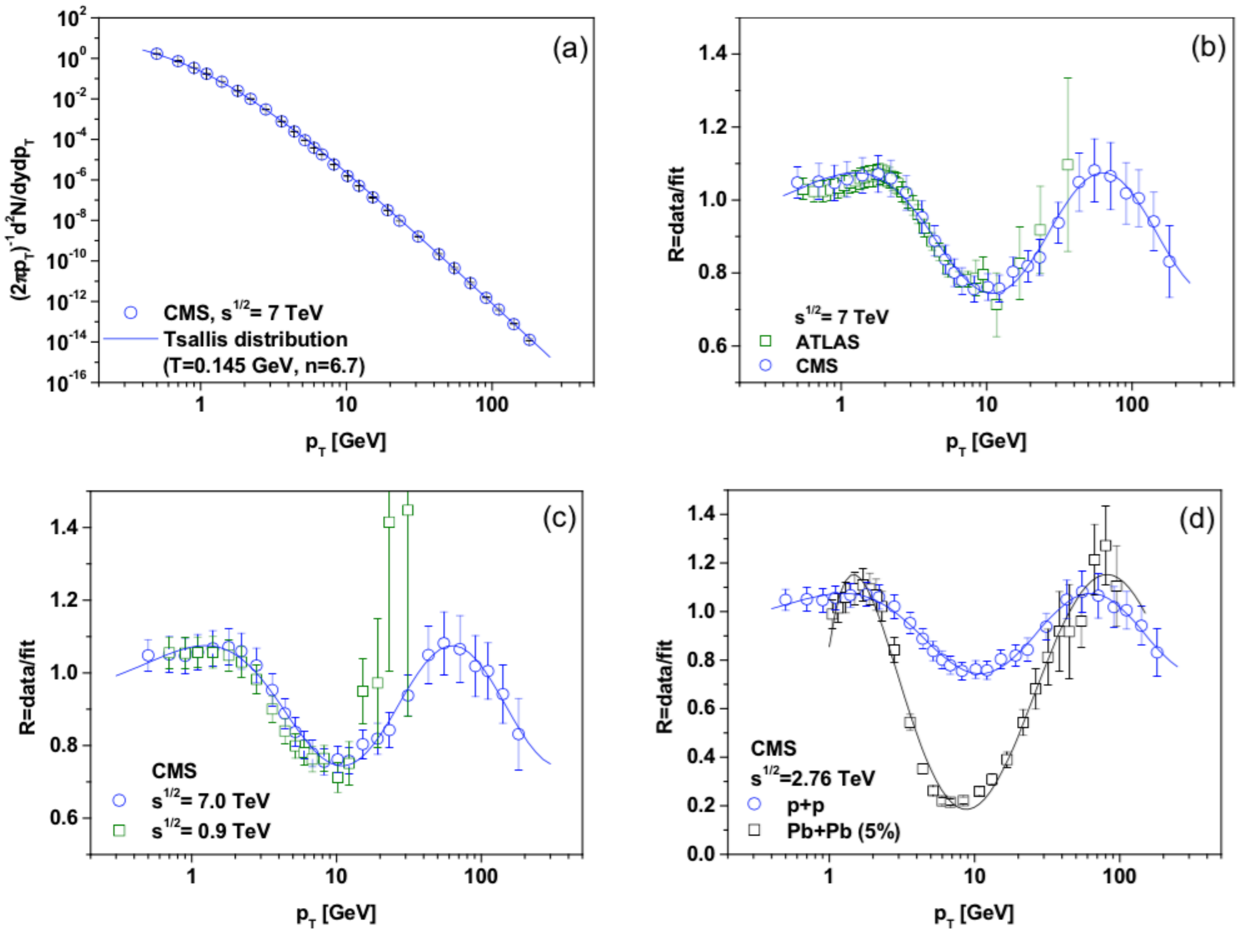
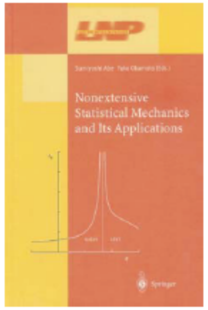


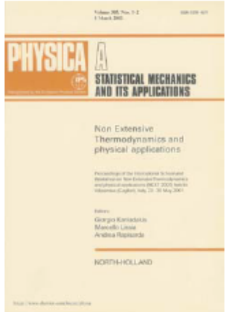
Figure 1. Examples of log-periodic oscillations. (**a**) dN/dp_T for the highest energy 7 TeV; the Tsallis behavior is evident. Only data from CMS experiment are shown [12]; others behave essentially in an identical manner. (**b**) Log-periodic oscillations showing up in different experimental data, like CMS [12] or ATLAS[15], taken at 7 TeV. (**c**) Results from CMS [12] for different energies. (**d**) Results for different systems ($p+p$ collisions compared with $Pb+Pb$ taken for 5% centrality [54]. Results from ALICE[55] are very similar. Fits for $p+p$ collision at 7, 2.76 and 0.9 TeV are performed with $q = 1.139 + i \cdot 0.0385$, $1.134 + i \cdot 0.0269$ and $1.117 + i \cdot 0.0307$, respectively. The fit for central $Pb+Pb$ collisions at 2.76 TeV is done with $q = 1.135 + i \cdot 0.0321$. See the text for more details.

BOOKS AND SPECIAL ISSUES ON NONEXTENSIVE STATISTICAL MECHANICS

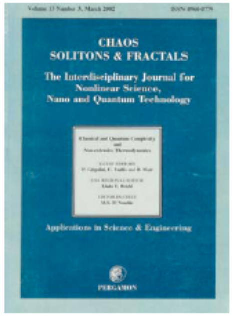
1999



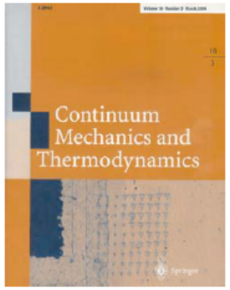
2001



2002

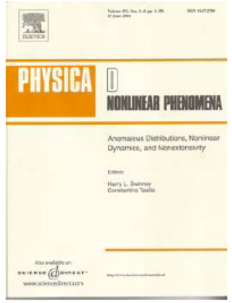


2002



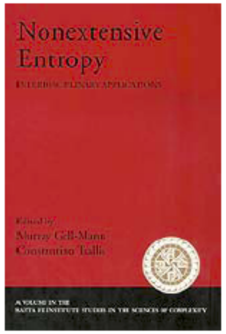
2004

2004

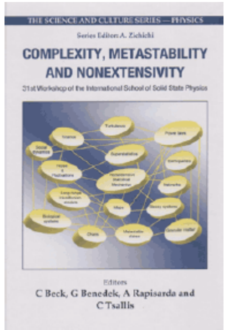


2004

2004



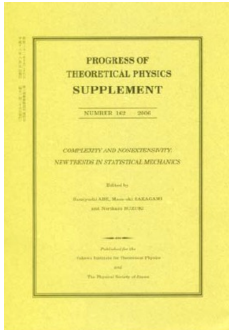
2004



2005

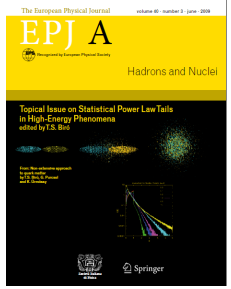
2005

2006



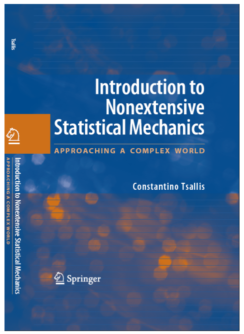
2006

2007

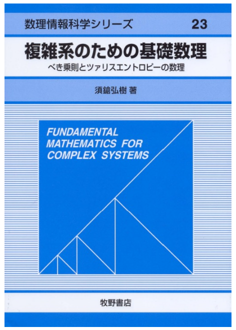


2009

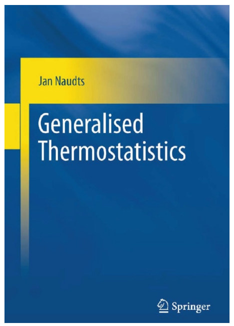
2009



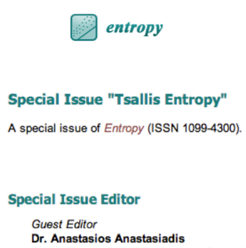
2009



2010



2011


entropy

Special Issue "Tsallis Entropy"

A special issue of *Entropy* (ISSN 1099-4300).

Special Issue Editor

Guest Editor
Dr. Anastasios Anastasiadis

2011

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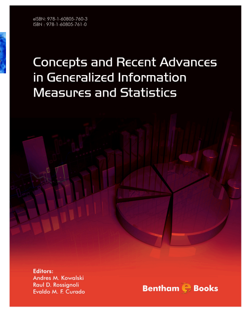

2013